



LEAN
SOLUTIONS
ACADEMY

A photograph of three business professionals in an office setting. A woman in a white blouse and black scarf is seated on the left, smiling. Another woman in a light blue shirt is leaning over her from the center, also smiling. A man in a light blue shirt is seated on the right, looking towards the women. The background is a blurred office environment. The image is framed by blue geometric shapes.

Lean Six Sigma Training **Black Belt**

brought to you by:



INTRODUCTIONS

TIMING and BREAKS

FIRE PROCEDURE

FACILITIES

SPECIAL REQUESTS

The ILSSI Exam

- Done online at ILSSI.org after training is finished
- Login and Password will be provided <https://ilssi.org/login/>
- Proctored by Webcam ([ProctorU.com](https://proctoru.com/))
- Sign-in and Schedule exam on [ProctorU.com](https://proctoru.com/)
- Open Book
- 100 Multiple Choice Questions
- Pass mark is 70%
- Result issued immediately
- Retake is available

ilssi.org



Body of Knowledge

1. **Roles and Responsibilities of Black Belt**
2. **Strategic Planning : Hoshin Kanri**
3. **DMAIC for Black Belts**
4. **Design for Six Sigma DFSS**
5. **Inferential Statistics / Central Limit Theorem / Standard Error**
6. **Advanced Hypothesis Testing (Continuous Data)**
7. **Advanced Hypothesis Testing (Attribute Data)**
8. **Advanced Design of Experiments (DOE)**
9. **Binomial and Poisson Distributions**
10. **Measurement System Analysis for Attribute Data**

Class Schedule

Day 1

- Black Belt Skills and Behavior
- Hoshin Kanri – Strategic Planning
- DMAIC Project Management
- Design for Six Sigma DFSS

Day 2

- Advanced Hypothesis Testing
- Advanced DOE
- Poisson and Binomial

Day 3

- MSA for Attribute Data
- **Black Belt Exam**



The Roles and Responsibilities

OF A BLACK BELT

What is the Role of a Black Belt?

*'There are **four purposes** of improvement:*

- Easier
- Better
- Faster
- Cheaper

These four goals appear in order of priority. Hence the first is to make the work easier for workers while improving the fruits of their labour.'

What is the Philosophy of a Black Belt?

Design and Develop the correct Products and Services

- Build Quality into the Design (DFSS / Quality Planning)

Build Efficient Processes for Delivering the Product

- Constantly improve them
- (Quality Improvement with DMAIC and Kaizen)

Set up Controls to ensure Stability and Quality

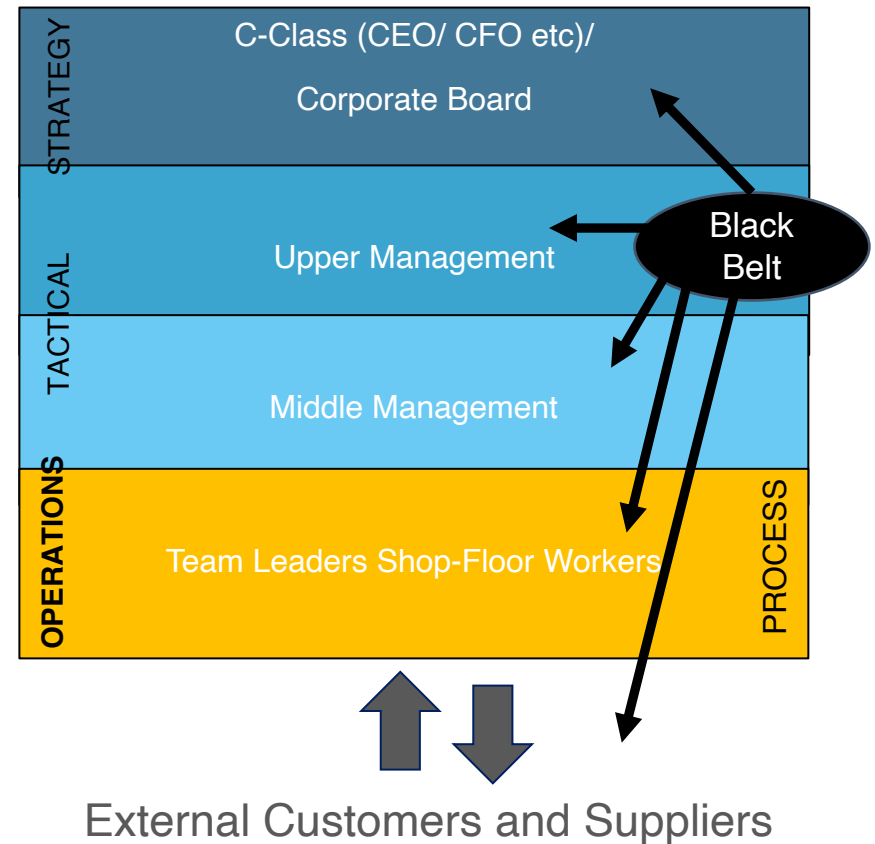
- (Quality Control)

What is the Philosophy of a Black Belt?



The Blackbelt's Place in Business

- Model of an organisation showing where Black Belts are considered in terms of **INFLUENCE** and **DECISION MAKING**.
- The BB should also have a dialogue with **CUSTOMERS** and **SUPPLIERS**

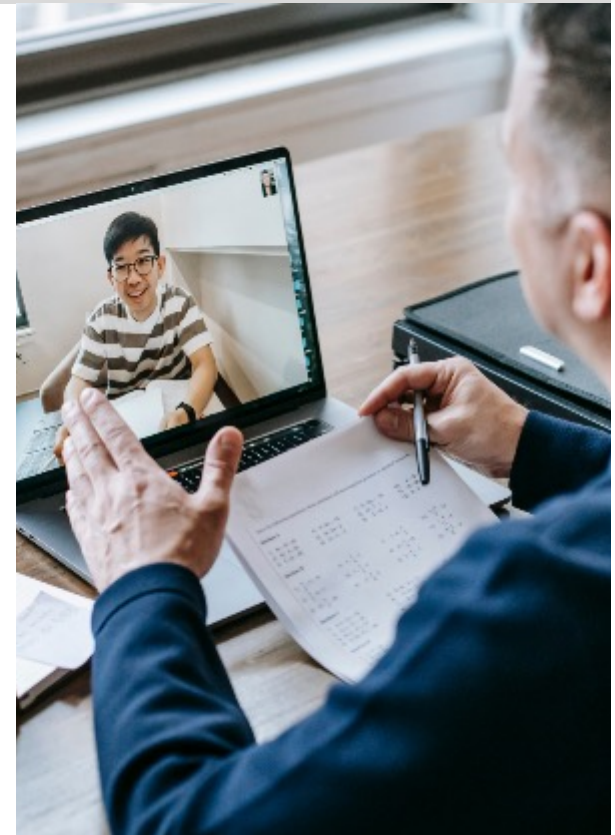


How Blackbelt Leaders Should ‘Think’

- Observe work – “go see” when problems occur and make decisions on facts
- Think and act long term
- Focus on Continuous Improvement
- Challenge management practices and learn new things
- Be humble with little interest in special privileges or status
- Respect and develop people at every level
- Put customers views first
- Believe that lean management is there to improve the entire business
- Soft skills and hard skills are connected and should work together

Blackbelt Skillset

- Leadership skills
- Problem Solving Skills
- Strategic Thinking
- Mentoring, Coaching & Training
- Project Management skills for large projects / programmes
- Practical understanding in use of Statistical tools
- Stakeholder Management
- Risk Management
- Change Management



Blackbelt Consults

Task #1: The Pitch

The Chief Operating Officer (COO) of an investment bank wants to cut the costs in the process for lending high value amounts to new clients. They want to use Lean Six Sigma to achieve the cost cutting targets and asks for your advice.

Give:

1. Two reasons why using Lean Six Sigma could be an advantage.
2. Two challenges that lie in using Lean Six Sigma to achieve the objective (reasons why the initiative might fail).
3. What are some of the first steps to take to implement a Lean Six Sigma programme ?



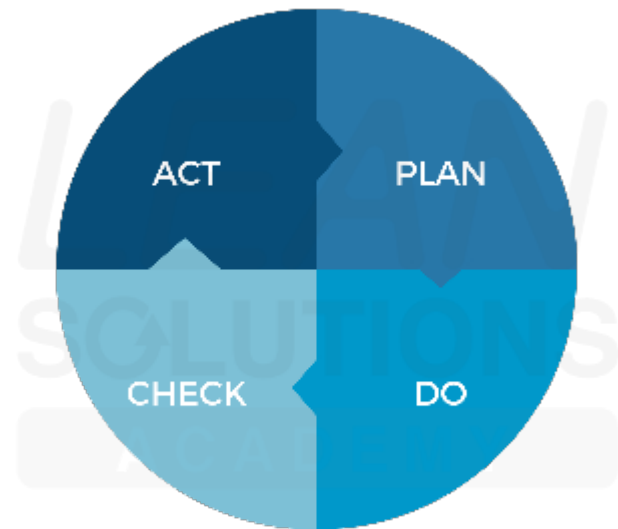
Strategic

PLANNING & DEPLOYMENT

Brief History

Hoshin Kanri was developed in Japan during the 1960s from Quality Management practices in:

- Bridgestone Tire company,
- Toyota,
- Denso,
- Komatsu
- Matsushita (Panasonic).



It was strongly influenced by the [Plan-Do-Check-Act \(PDCA cycle\)](#) of W. Edwards Deming, and Management by Objectives of Peter Drucker.

Brief History

Hoshin = “The Compass” or “Direction Setting”



Kanri = Control with a **Logical Plan** (“The Map”)



“A statement of desired outcome for the future, plus the means of accomplishing that outcome, and for measuring the accomplishment.”

Strategic Principles



CONTINUOUS
IMPROVEMENT

The Hard Stuff

Belted Training / Measure
Projects etc...



RESPECT &
DEVELOPMENT
OF PEOPLE

The Soft Stuff

Behaviours, Motivation
How People Think

Strategic Principles

FAKE LEAN



CONTINUOUS
IMPROVEMENT

We Focus on Just CI

SUSTAINABLE LEAN



CONTINUOUS
IMPROVEMENT



RESPECT &
DEVELOPMENT
OF PEOPLE

We Put More Focus on Respect
and Development of People

Strategic Long-Term Objectives

Process
Improvement
& Innovation

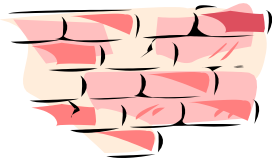
Value Creation
for Customers

Workplace
Balance
& Harmony

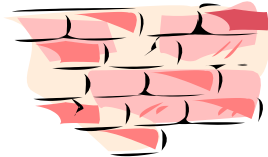
Financial
Sustainable
Improvement

Strategic Principles

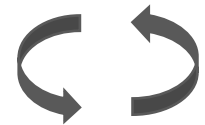
Starting Point



Fake Lean



True Lean



OPERATIONS



Added Value – Lean Behaviors



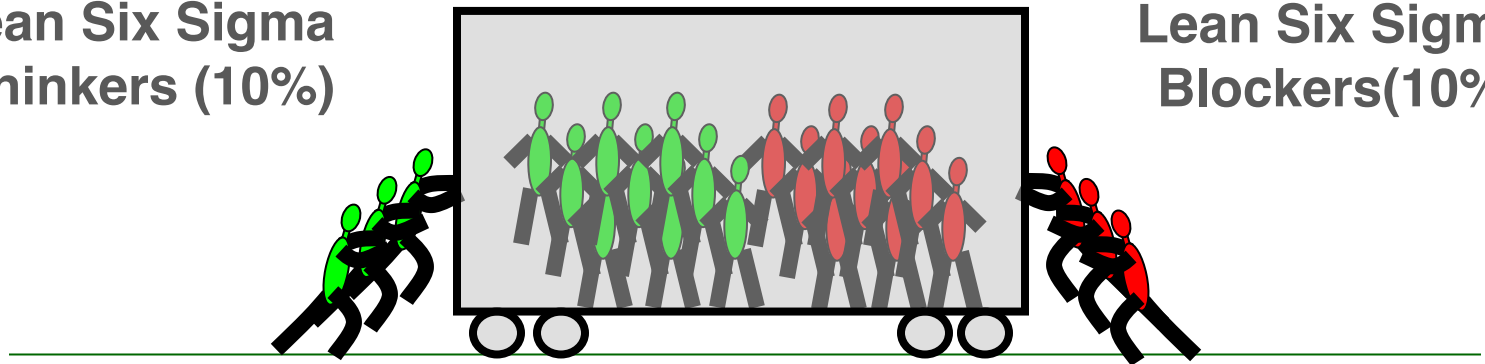
A Typical Workforce...

Willing to
Support (40%)

**Lean Six Sigma
Thinkers (10%)**

Quick to
Criticize (40%)

**Lean Six Sigma
Blockers (10%)**



The direction you want to go

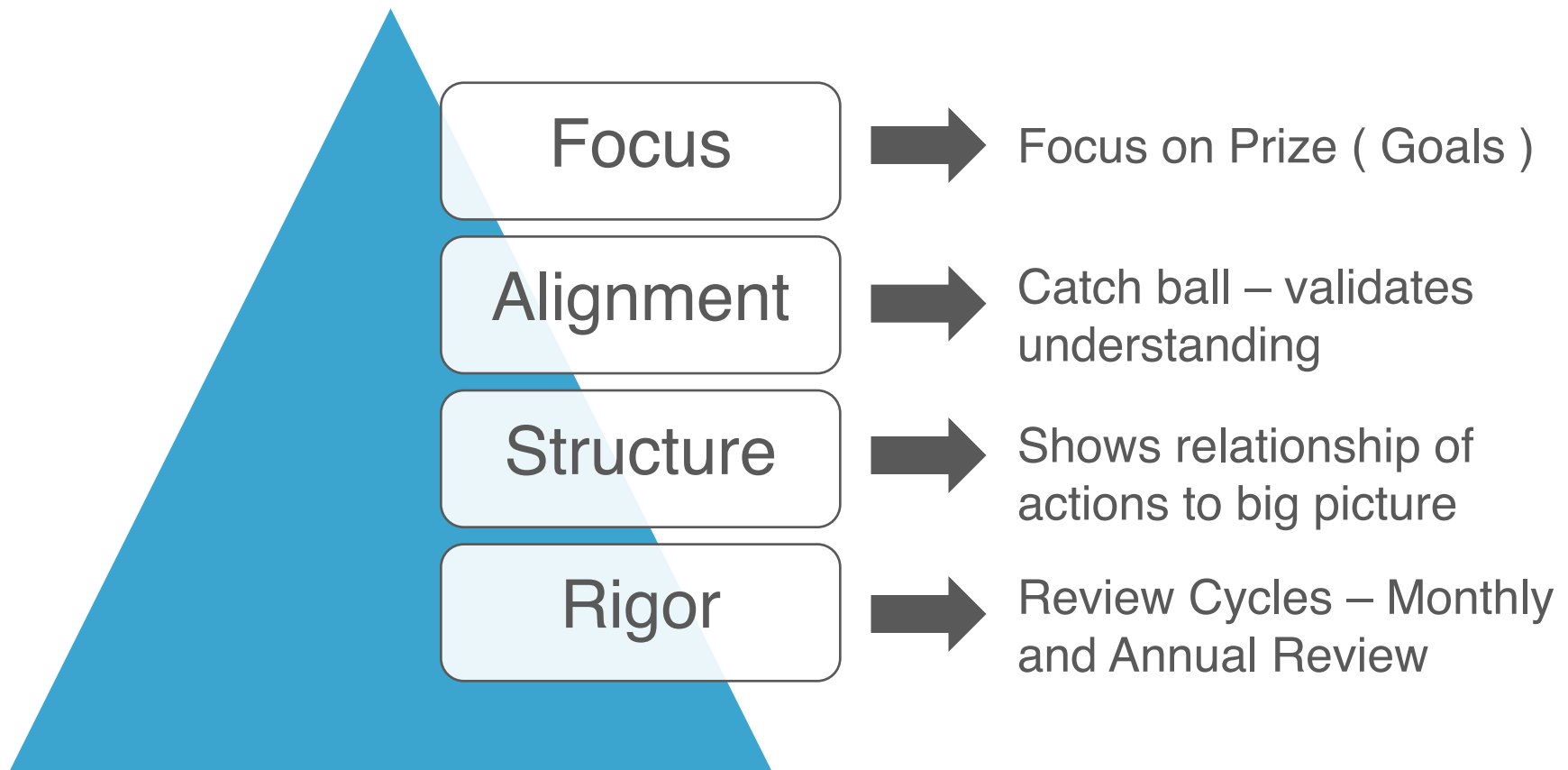
Keep on Pushing

What is Organizational Culture?

“ The way we do things around here”

- The beliefs, attitudes and values that exist within your organisation.
- Every company will have factors that influence its company culture.

Four Elements of Hoshin Kanri



Hoshin Planning Process

C
A
T
C
H
B
A
L
L

PLANNING



1. Establish Vision / Mission for Business
2. Develop 3 to 5 Year Strategic Objectives
3. Develop Annual Objectives and Improvement projects
4. Prioritize and Schedule Improvement Projects

IMPLEMENTATION

Lean SS
Projects

5. Implement Improvement projects

REVIEW



6. Monthly Review
7. Annual Review

Vision / Mission / Values

Vision

- An aspirational statement illustrating how the organisation will look in the future and how it will benefit if the change happens

Mission

- A statement defining what an organization is, why it exists, its reason for being.

Values

- Fundamental beliefs of the organization. These guiding principles dictate behavior and help employees understand the difference between right and wrong.

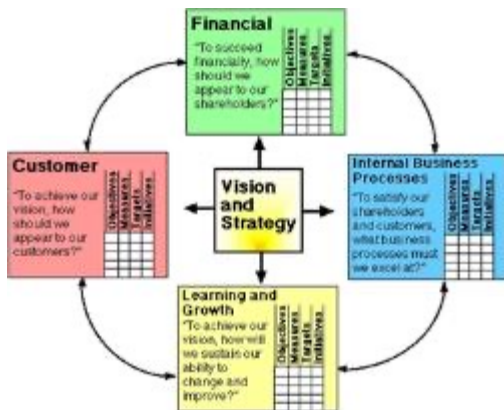


“A Black Belt must be committed to the company Vision, Mission and Values”

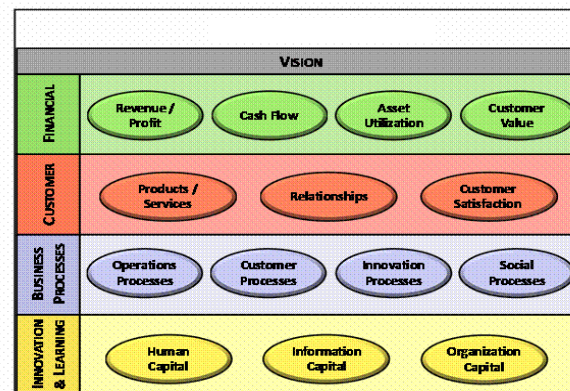
Develop Strategic Objectives

Strategic Planning is determining those 3 to 5 year breakthrough objectives that will move the organization towards world class.

Balanced Score Card



Strategy Maps



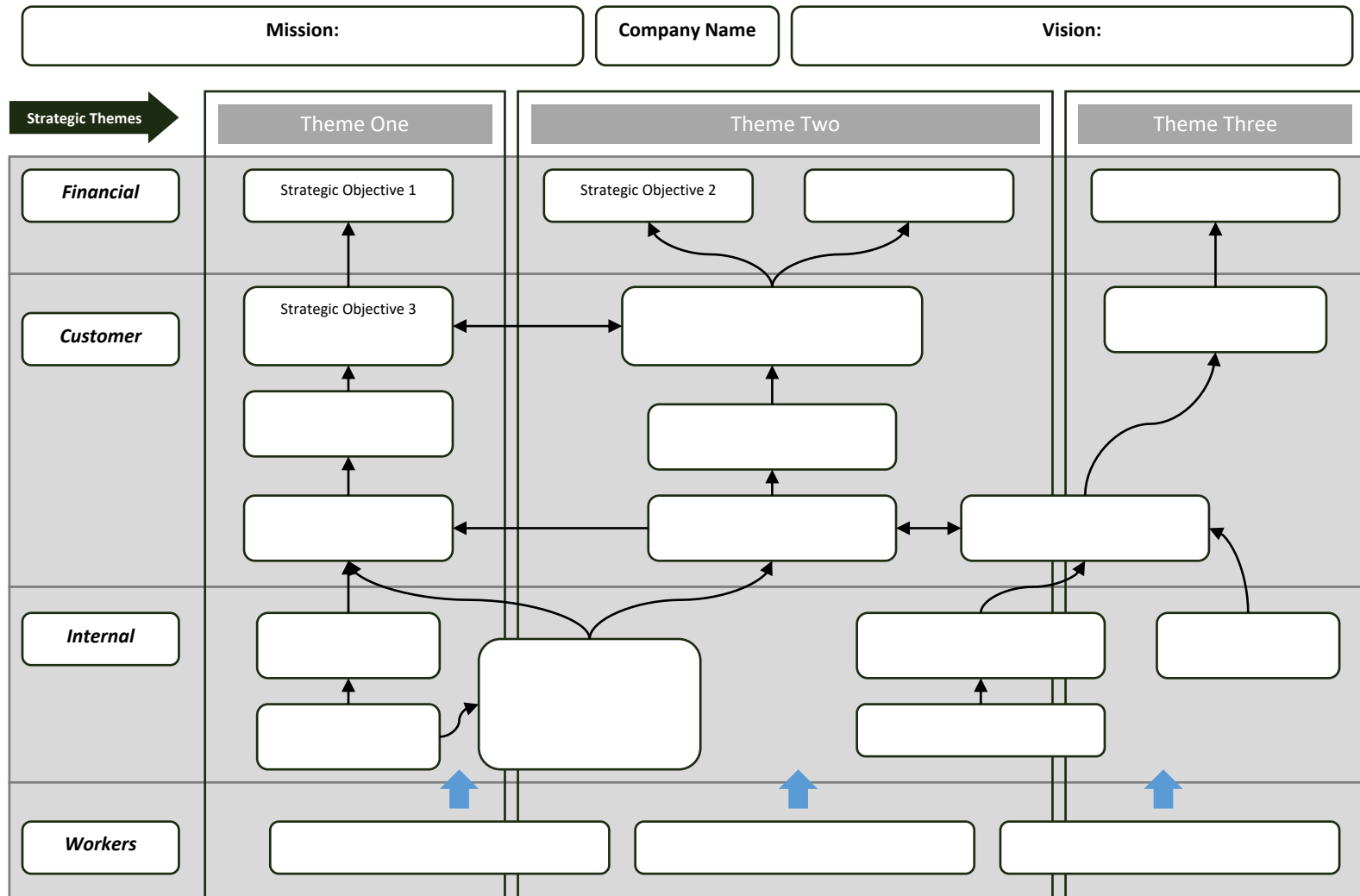
SWOT Analysis



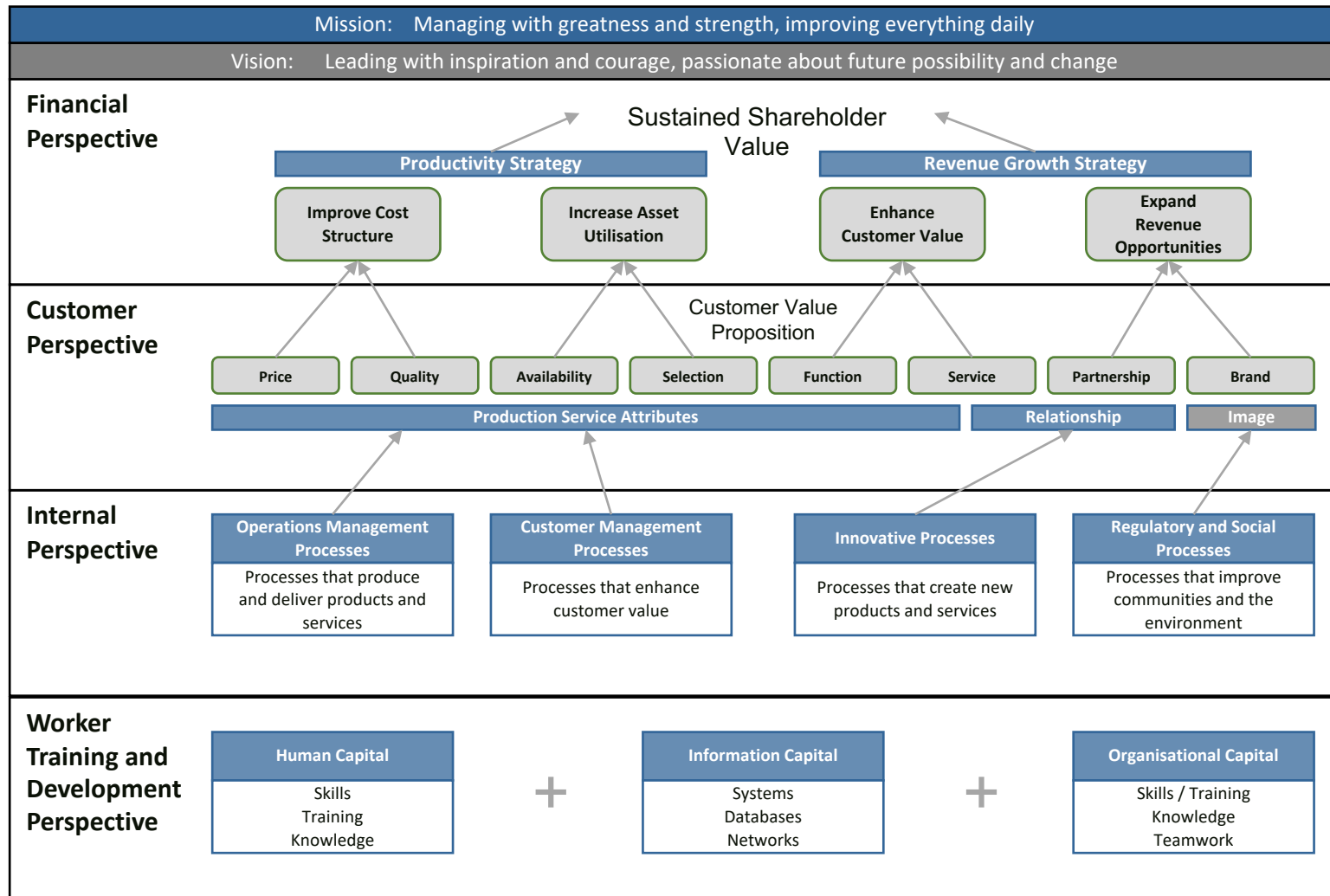
Basic Strategy Map

Financial	
Customer	
Processes	
Workers	

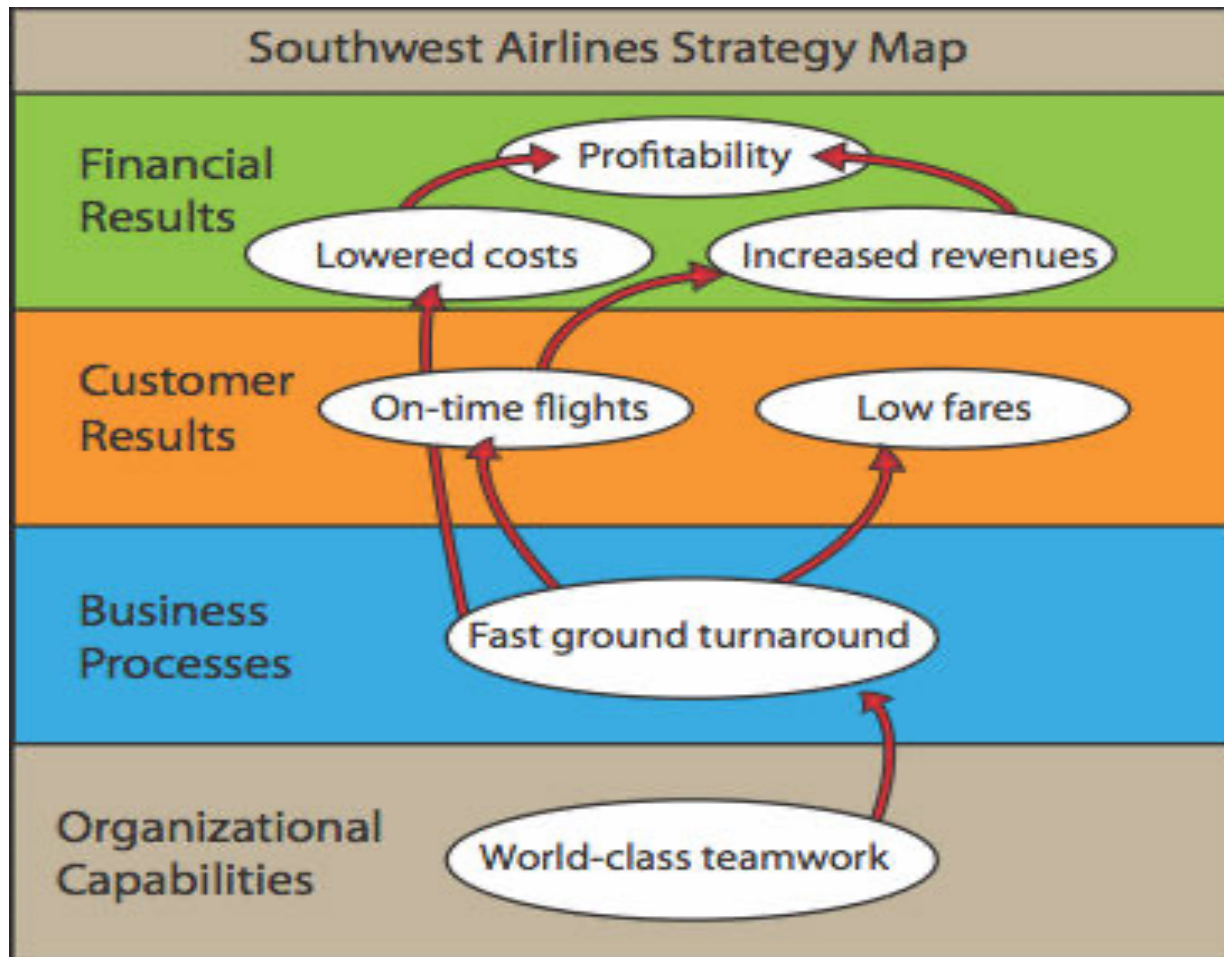
Add More Detail



Strategy Man – Generic Example



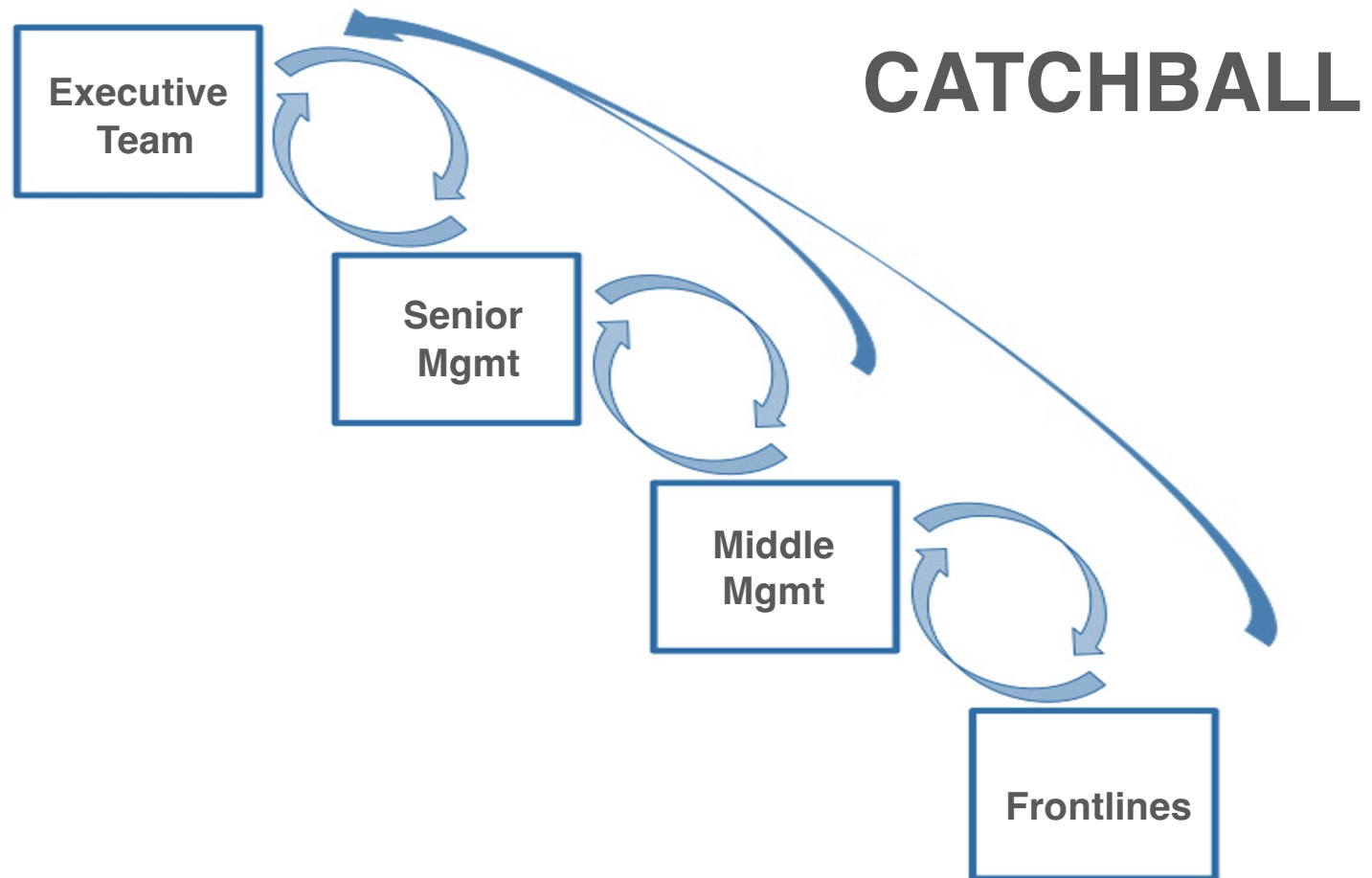
Strategy Man – Specific Example



Catch – Ball Process



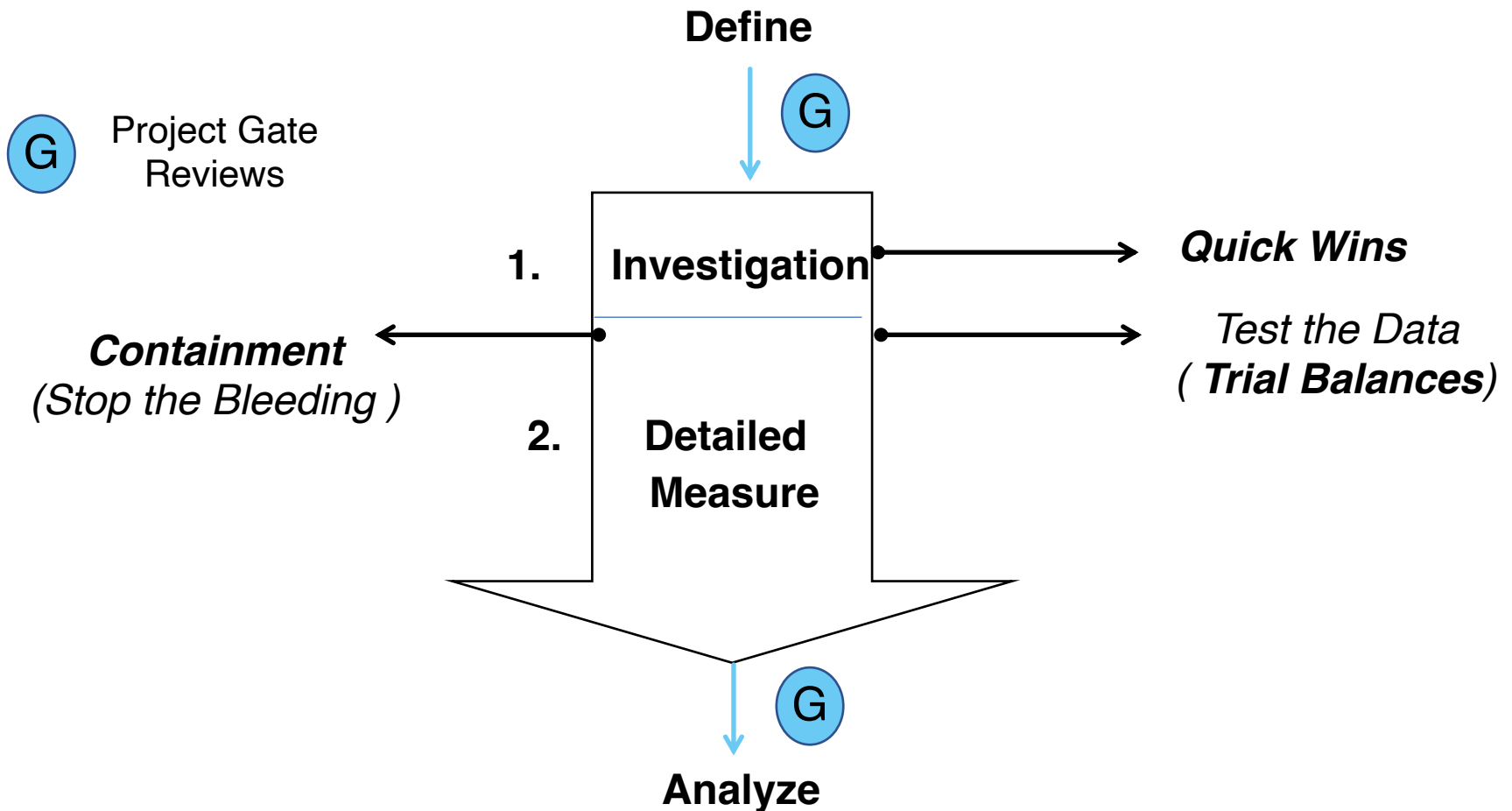
Strategy Deployment



DMAIC Projects

FOR BLACK BELTS

Two Step Measure



Trial Balances, Quick Wins, & Containment Actions

Trial Balances

- Try analysing your data early on (days/weeks) – you will not have enough data to identify root causes but you will be able to see if anything is missing

Quick Wins

- Just by measuring something you often suddenly see something obvious to fix – this might reduce the problem or give another benefit.
JUST DO IT THERE AND THEN !

Containment

- A 'Work Around' to prevent further problems
- A 'Cause' of the problem is identified and a 'fix' put in place
- Often an interim solution.
- 'Stopping the Bleeding'

Project Portfolio Management

Project portfolio management (PPM) is a key skill of any Black Belt with accountability for multiple projects. (also called '*Programme Management*')



Control



Efficiency



Visibility



Governance

Efficiency and Control

Use a Project Portfolio Management (PPM) tool for better
Efficiency and Control

Companion by Minitab

<http://www.minitab.com/en-us/products/companion>

COMPASS

<https://www.creato.com/compass-qms>

Microsoft Project Server 2016 / Project Portfolio Management (PPM)

<https://products.office.com/en-gb/project/enterprise-project-server>

Managing Stakeholder Expectations

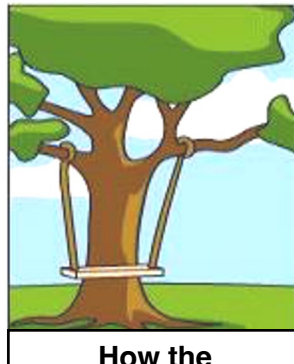
Black Belt Governance duties at the programme level:

- Selection of the correct Projects
- Alignment of Projects with Company Vision (Hoshin Kanri)
- Gate Reviews
- Status Reports to Upper Management / Executives
- Risk Management of portfolio / programme
- Change Management for all projects

Managing Stakeholder Expectations



How the customer explained it



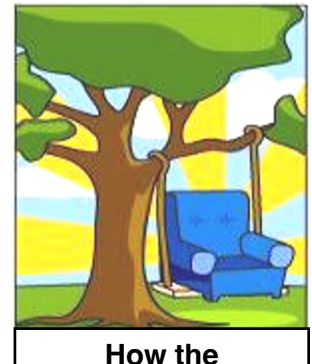
How the project leader understood it



How the analyst designed it



How the programmer wrote it



How the consultant described it



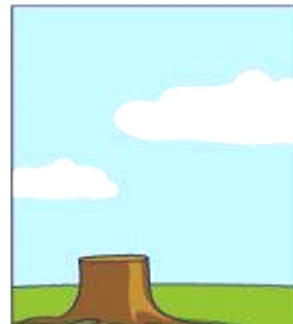
How the project was documented



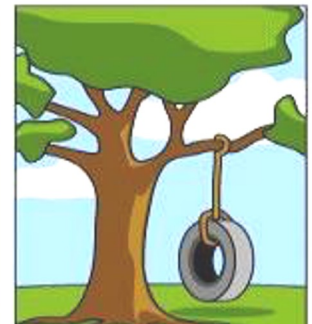
What was installed by operations



How the customer was billed



How the project was supported



What the customer really needed

Change Management

- **Requests for Change** (RFCs) are written proposals for change to any existing process
- The BB needs to make sure that there is a solid Change Management process for all Lean Six Sigma projects
- A **Change Control Board** (CCB) is often used as part of the Change Management Process for Approval / Rejection of RFCs
 - For more information of RFC, CCB and Change Management see:
 - <https://alison.com/courses/introduction-to-change-management/content>



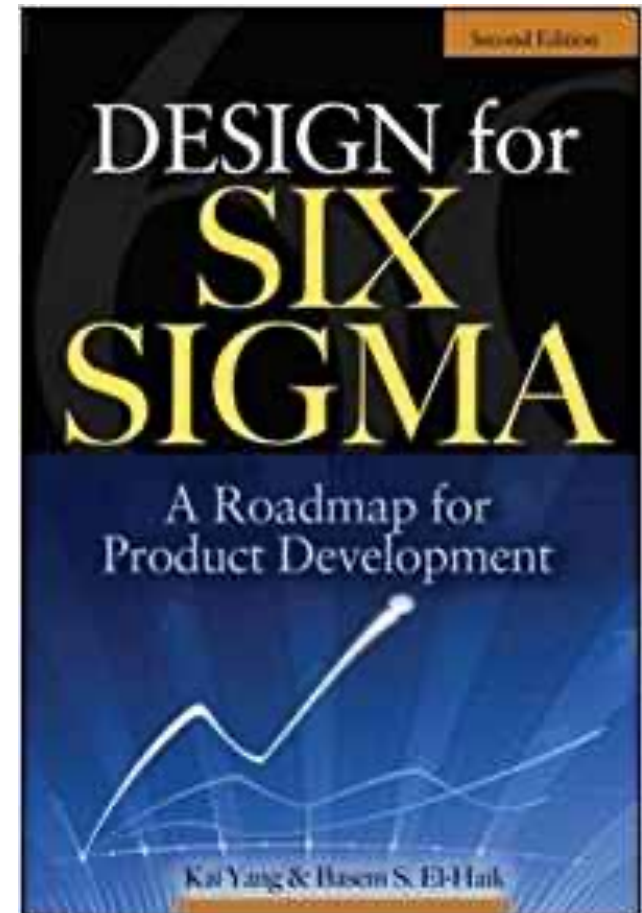
Design For

Six Sigma DFSS

Literature

Design for Six Sigma: A Roadmap for Product Development

2008 - By [Kai Yang](#) [Basem Ei-Haik](#)



Why Use Design for Six Sigma?

Design For Lean Six Sigma because it...

- Reduces the number of **DEFECTS** delivered to our customers
- Reduces the **REWORK and SCRAP** in the production of our products, processes or services
- Reduces the **LEAD TIME** in the delivery of our products, processes or services
- Reduces **COST** of production and delivery of our products, processes or services

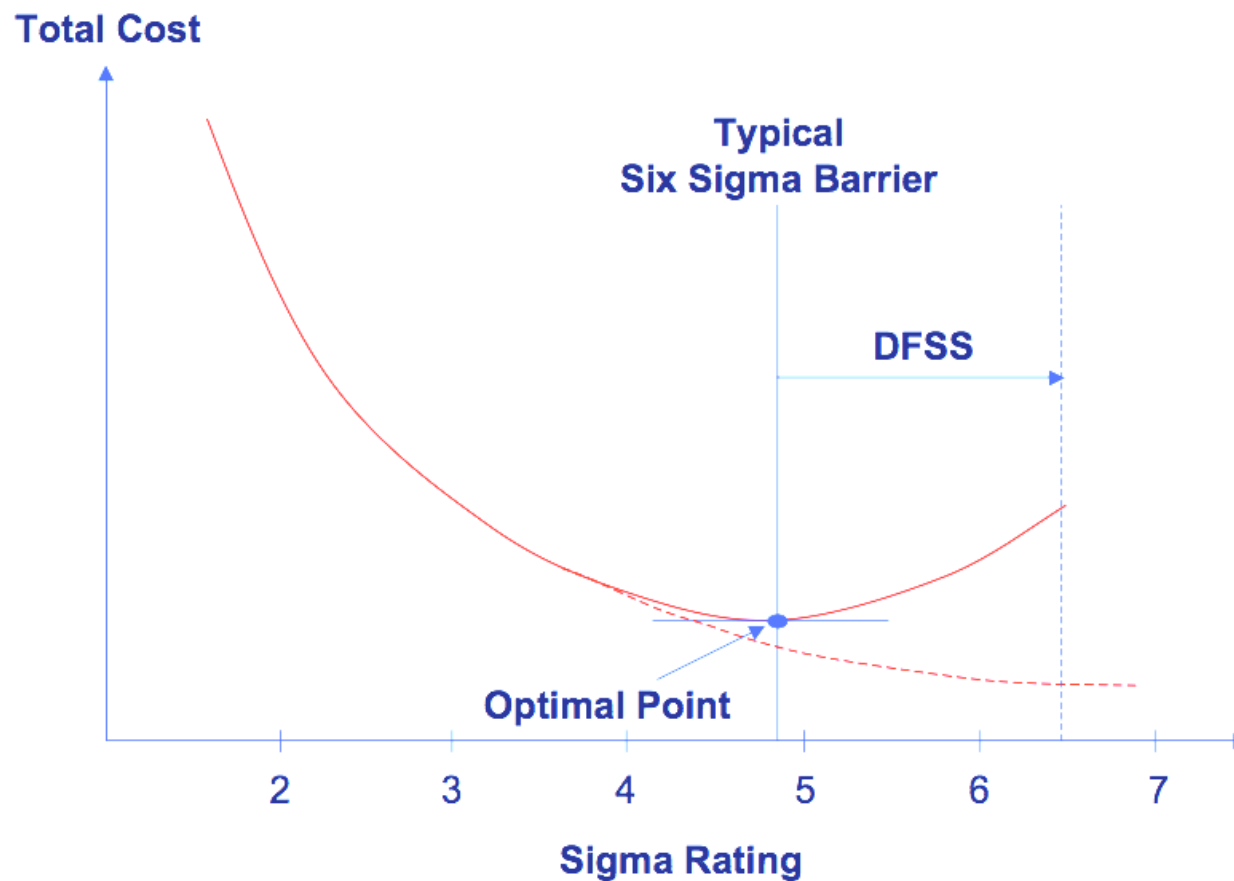
DFLSS to Reduce the Risk of New Product Launches

- Launching new products, processes and services has historically been a high risk activity.
- More than 85 % of new Fast Moving Consumer Goods (FMCG) products that established companies put into the market fail to deliver the expected return on investment.

(Neilson Group 2014)

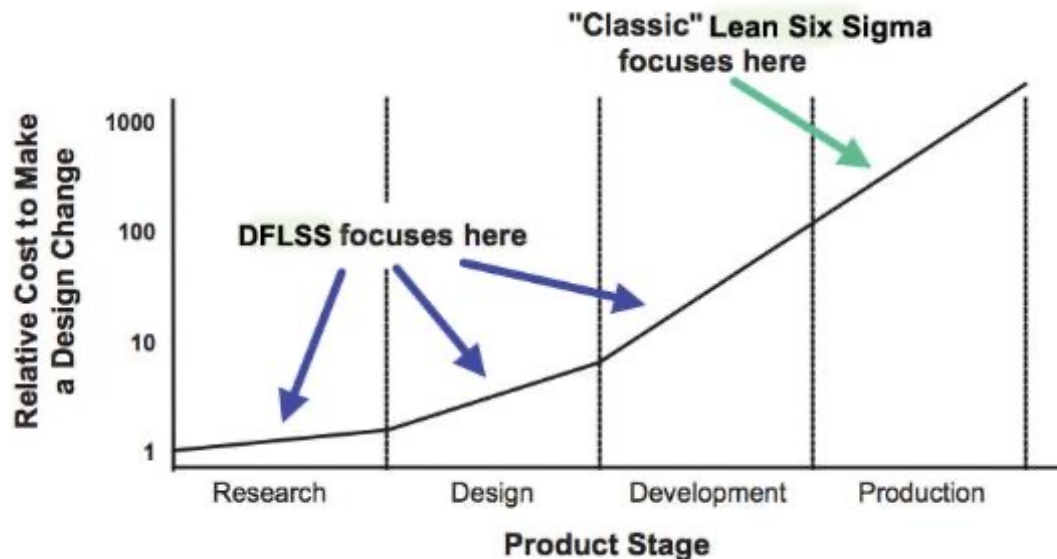
www.nielsen.com/us/en/insights/news/2014/how-to-flip-85-misses-to85-hits-lessons-from-the-nielsen-breakthrough-innovation-project.html

DFLSS to Break the 5 Sigma Barrier



When to Use DFLSS

DFSS is always used EARLY in the new product development lifecycle.

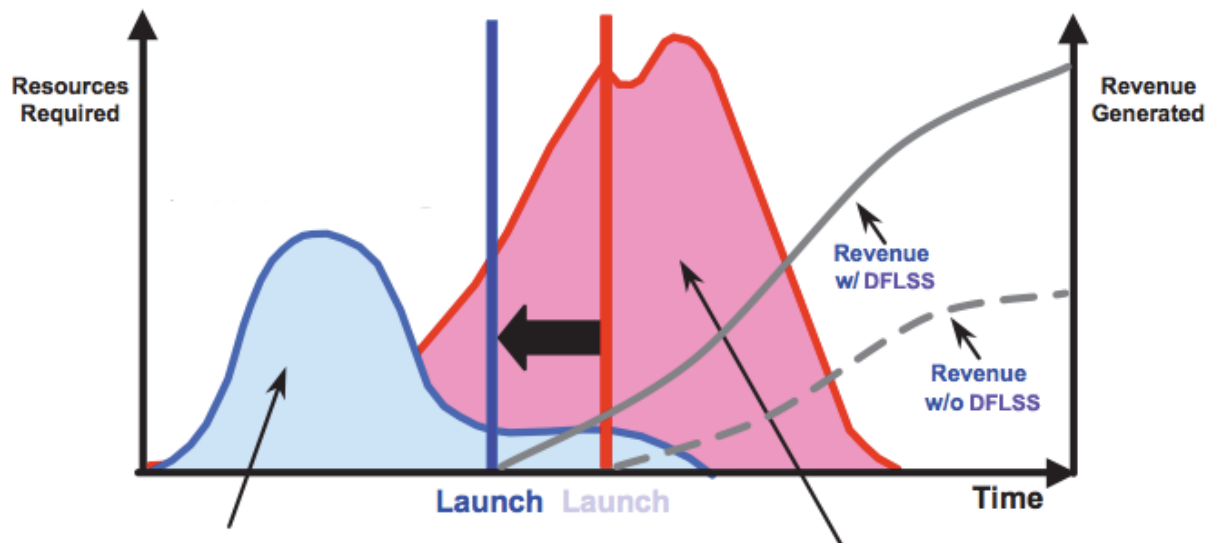


Why?

Because Cost of Changes increases with time / stage in product lifecycle.

DFLSS: Predictive vs Reactive Design

Front-End Loading of Resources for Predictive Design:



With DFLSS : Predictive Design

- Early problem identification; solution when costs low
- Faster market entry: earlier revenue stream, longer patent coverage
- Lower total development cost
- Robust product at market entry: delighted customers

Without DFLSS : Reactive Design

- Unhappy customers
- Unplanned resource drain
- Skyrocketing costs
- Next product compromised

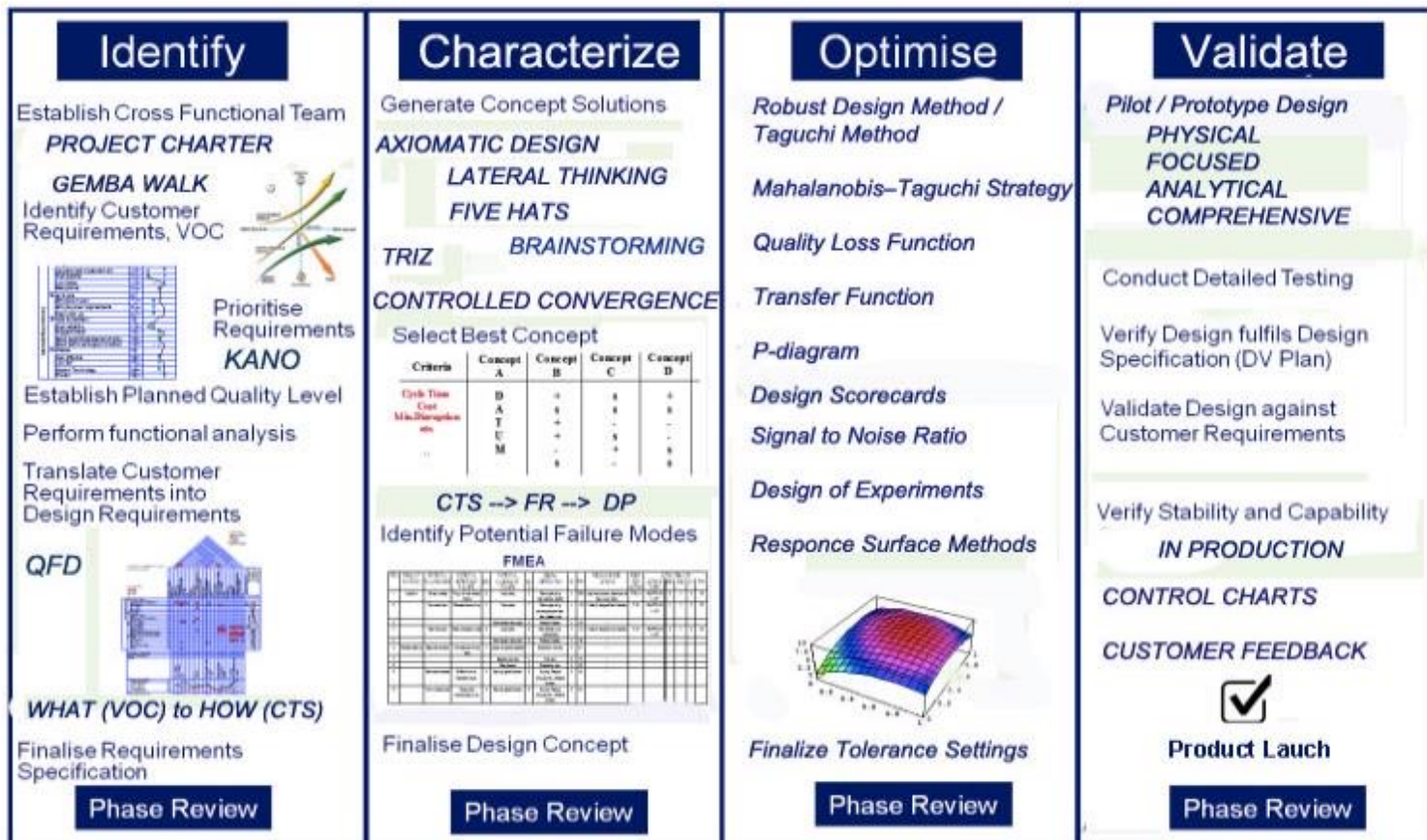
DFLSS Deployment Methodologies (Design Algorithms)

Several Popular Methodologies (Design Algorithms):

- **DMADV** (Define, Measure, Analyse, Design, Validate).
- **ICOV** (Identify, Characterize, Optimize, Validate)
- **IDOV** (Identify, Design, Optimize, Validate).
- **I2DOV** (Invent / Innovate, Develop, Optimize, Validate).
- **CDOV** (Concept, Design, Optimize, Validate).

*Note: All approaches use the same philosophical approach.
It is only the vocabulary that is used that is different.*

ICOV Methodology



Steps of Methodology - Roadmap

Step 1

Identify

- **FORM TEAM**
- **DEFINE / IDENTIFY / UNDERSTAND** your customer, Understand your market, Understand their needs and wants. Understand what **FUNCTIONAL REQUIREMENTS**.
- **(FR)** are important to them. Understand the competition. Understand the environment. Survey and Measure to confirm all understanding.

Step 2

Characterize

- **INNOVATE / CONCEPTUALIZE / CHARACTERIZE** a new or improved product or service which will satisfy FR to satisfaction of customers. **SELECT** best option.
- **DESIGN and DEVELOP** the concept into a working model.

Step 3

Optimize

- **OPTIMIZE** the new / improved model by experiment, measurement and analysis. Adjust tolerances to make the model as robust and free of potential defects as possible.

Step 4

Validate

- Prototype / Pilot the optimized working model to **VALIDATE / VERIFY** that it meets the demands of the customer. Test and measure capability of the new/improved design to meet the expectations of customers. Release into production and continue to monitor.

Quality Function Deployment - QFD

The House of Quality

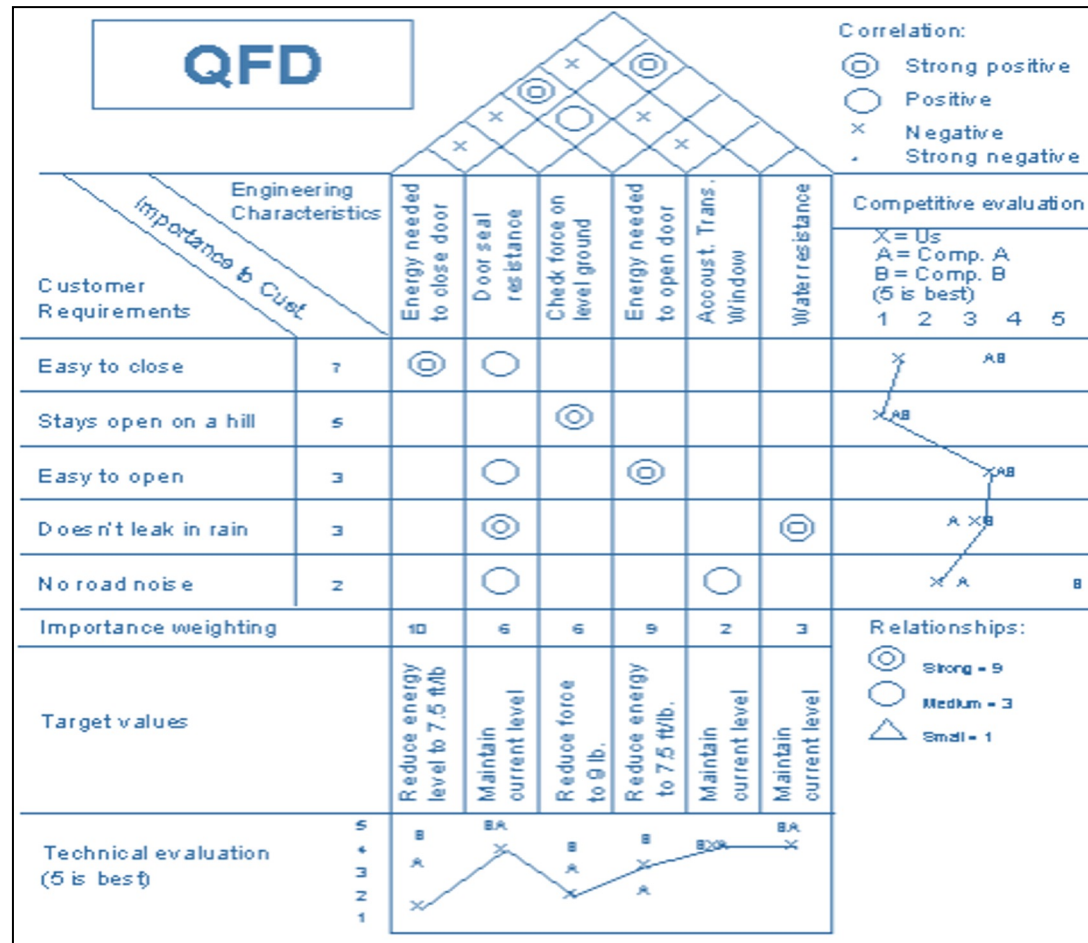


Quality Function Deployment Overview

- Quality Function Deployment (QFD) is a structured approach to understanding true customer priorities and a business's ability to fulfil them
- QFD starts by identifying customer needs, wants, and wishes then translates them into the features and functionality of the product or service to meet those needs
- It can be used when-ever you are designing or developing a new process where customer satisfaction is key
- Typically uses a four step matrix based approach (House of Quality)



House of Quality



SigmaXL QFD



+

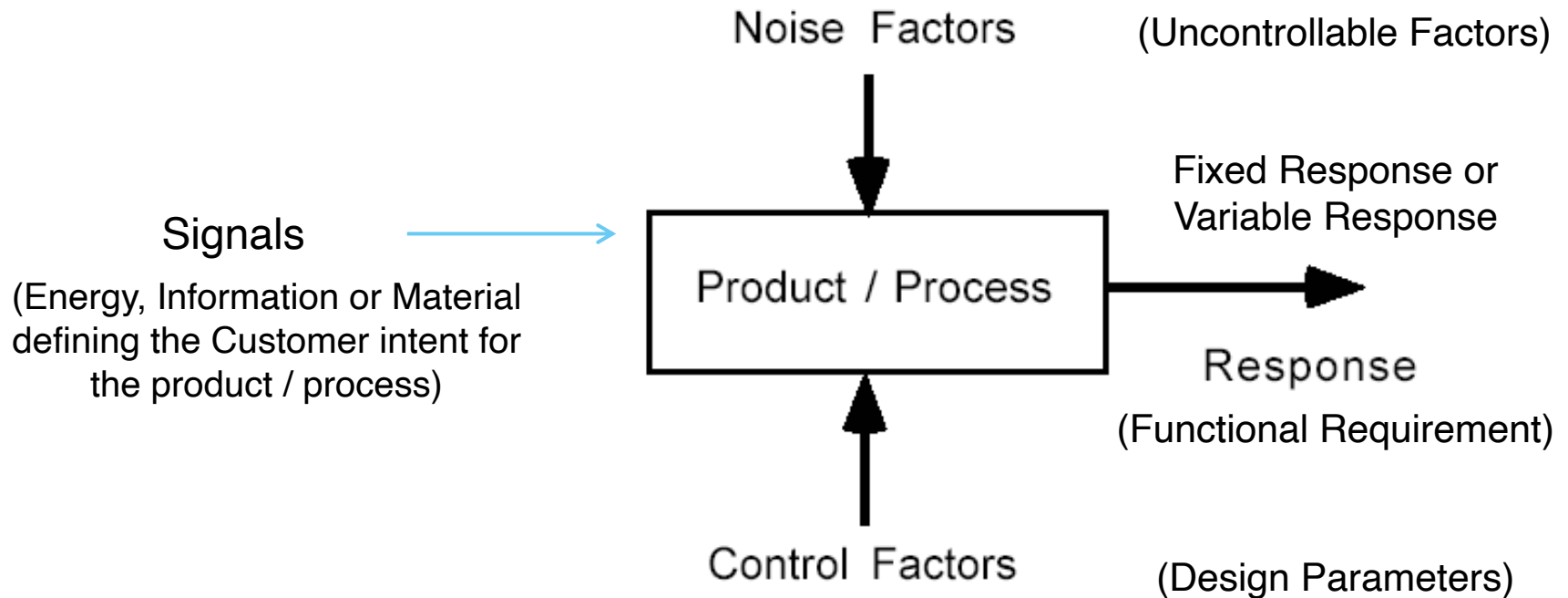
		+ + + + + + + + + + + + + + + + + + + + + +															
		↑ ○ ↑ ↑															
		Desired Direction of Improvement: Functional Requirements (How): Satisfaction Ratings:															
Customer Requirements (What):	Customer Importance Rating	Functional Requirement 1	Functional Requirement 2	Functional Requirement 3	Functional Requirement 4	Functional Requirement 5	Functional Requirement 6	Functional Requirement 7	Functional Requirement 8	Functional Requirement 9	Functional Requirement 10	Our Customer Satisfaction (CS) Rating	Competitor 1 Rating	Competitor 2 Rating	Competitor 3 Rating	Competitor 4 Rating	CS - Max(Competitor Rating)
Lightweight	4	9	3		9							3	2				1
Easy to use	5	9	9	9	9							4	4				0
Reliable	2	3	3	1	9							4	3				1
Easy to hold steady	3	9	1	1	9							1	3				-2
No double exposures																	0
Customer Requirement 6																	0
Customer Requirement 7																	0
Customer Requirement 8																	0
Customer Requirement 9																	0
Customer Requirement 10																	0
Raw Score:		114	66	50	126	0	0	0	0	0	0						
Rank:		2	3	4	1	5	5	5	5	5	5						

P-Diagrams

P-Diagram (Parameter Diagram of Process Diagram)

- A graphical tool to help visualize the design problem in DFSS methodology.
- **Provides a structured method to identify:**
 - Intended Customer Inputs (Signals) M
 - Unintended Inputs (Noise Factors) N
 - Design Controllable Factors X
 - Intended Outputs, Response (Ideal Function) Y

P-Diagrams



At least 1 Control Factor (DP) per Functional Requirement (FR)

Example - Refrigerator

- CTQ = Store fresh food in the home for both long term and short term use
- FR1 = Freeze food for long-term preservation
- FR2 = Maintain food at cold temperature for short term preservation

To satisfy these two FRs, a refrigerator with two compartments is designed.

Two DPs for this refrigerator may be stated as:

- DP1 = The freezer section
- DP2 = The chiller (refrigerator) section



Example of Decompose - Refrigerator

Having chosen the DP1, we can now decompose FR1 into secondary functional requirements as: **FR1**

FR11 = Control temperature of the freezer section in the range of -18 C +/- 2 C

FR12 = Maintain the uniform temperature throughout the freezer section at the preset temperature

FR13 = Control humidity of the freezer section to relative humidity of 50 %

Class Exercises

1. Fizzy Drink Can

- Name the Primary FRs for a Fizzy Drinks can
- Name the Primary DPs for a Fizzy Drinks can



2. A Umbrella

- Name the Primary FRs for a Umbrella
- Name the Primary DP's for a Umbrella



3. A Running Shoe

- Name the Primary FRs for a running shoe
- Name the Primary DPs for a running shoe



Robustness Design

Robustness means insensitivity to variations caused by noise factors, which may include environmental factors, user conditions, and manufacturing disturbances.

<https://www.youtube.com/watch?v=DiLD6ACw7fQ>

Shin Taguchi (son of Genichi Taguchi)

Cost vs Quality Trade Off

Taguchi Loss Function

Other Names Include:

- Taguchi Quadratic Loss Function
- Quality Loss Function

The Taguchi Loss Function translates variation and ‘Off Target’ production into a monetary value \$\$\$ lost in customer value.

Taguchi Loss Function

$$L = k * E (Y - T) ^ 2$$

Where

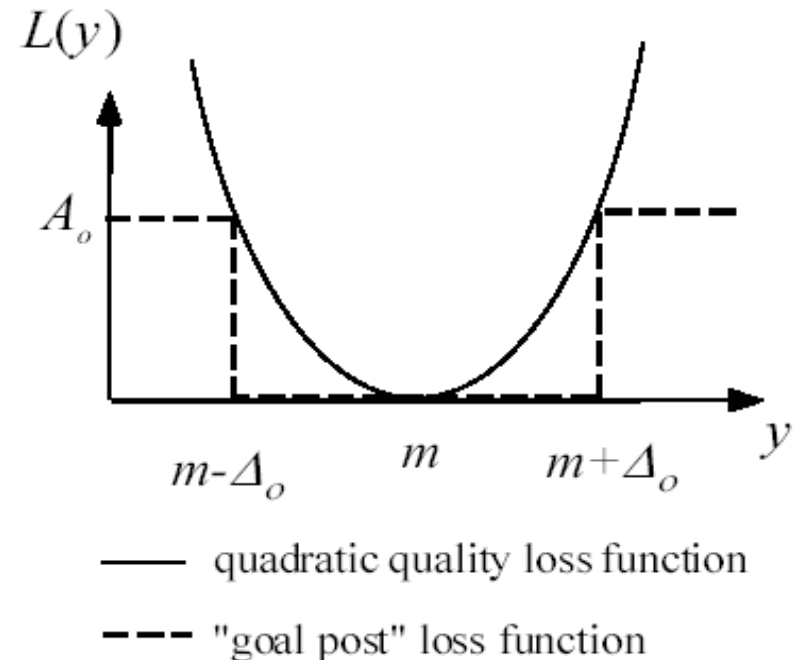
- L** = value loss (in \$\$)
- k** = quality loss coefficient
- E** = Expected value by customer
- T** = Target value by supplier
- Y** = Actual value

The objective of Taguchi Tolerance Design is to minimize L by finding the perfect balance between reduced tolerances and increased costs of production

Quality Loss Function

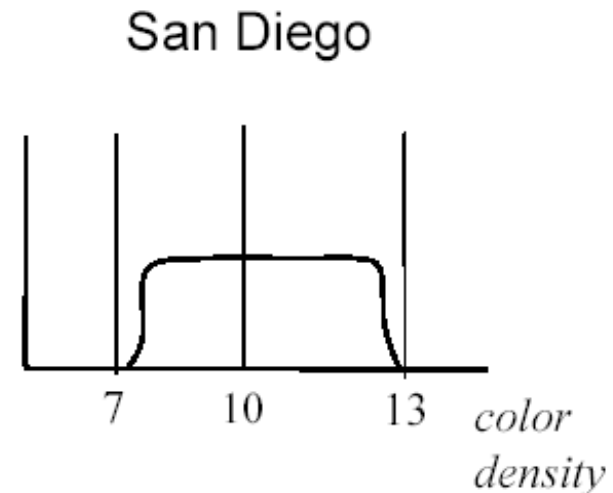
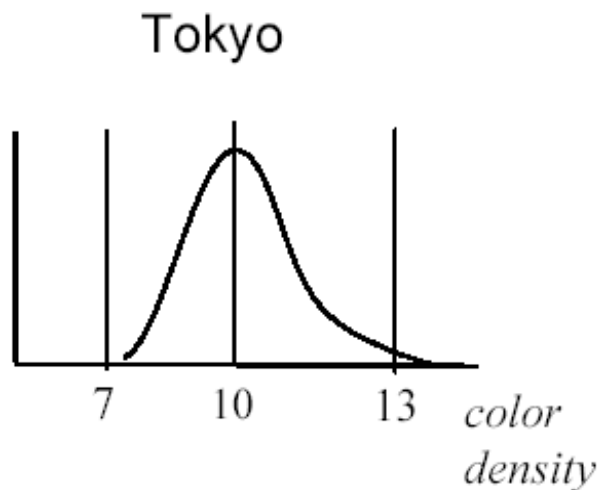
\$\$\$ Loss due to deviation from target quality

- Zero at the target value
- Equal to scrap cost (A) at anywhere outside the tolerance limits



Example – Sony Televisions

- Sony TVs manufactured in both JAPAN and USA
 - See: <https://statweb.stanford.edu/~owen/courses/363/lecture13.pdf>
- Distribution of color density varies around the mean
 - Which has lower defect rates?
 - Which one has better quality?



Inferential Statistics

The Central Limit Theorem

Standard Error of the Mean

Inferential Statistics Defined

Inference :

*A conclusion reached on the basis of evidence and reasoning.
(Oxford Dictionary)*

There are 2 goals in using Inferential Statistics :

1. To draw a conclusion about a population based on the analysis of sample data from the population.
2. To quantify the level of confidence that we have in our conclusion. To understand the level of risk that we are taking in acting upon our conclusion.

The Central Limit Theorem

The Central Limit Theorem says:

As the sample size becomes larger the distribution of means of the samples will form a Normal Distribution, the spread of the distribution becomes narrower, and the mean of the means will move towards the mean of the population.

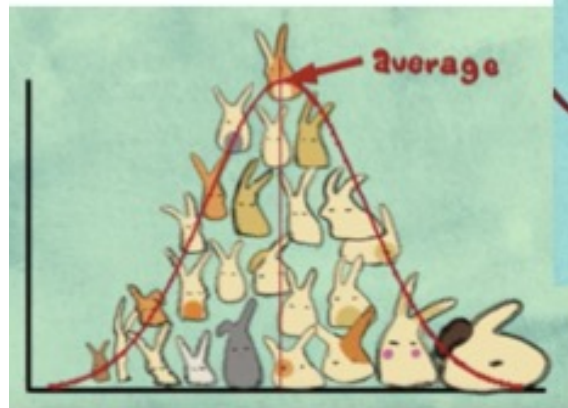
The Central Limit Theorem is key to Inferential Statistics

- The first version of this theorem was postulated by the French-born mathematician Abraham de Moivre who, in a remarkable article published in 1733, used the normal distribution to approximate the distribution of the number of heads resulting from many tosses of a fair coin.

The Central Limit Theorem

Bunnies, Dragons and the 'Normal' World: Central Limit Theorem | The New York Times

<https://www.youtube.com/watch?v=jvoxEYmQHNM>



Sampling Distributions – The Foundation of Statistics

Population

Samples from the population, each with five observations:

3
5
2
12
10
1
6
12
5
6
12
14
3
6
11
9
10
10
12

	<i>Sample 1</i>	<i>Sample 2</i>	<i>Sample 3</i>
	1	9	2
	12	8	3
	9	5	6
	7	14	11
	8	10	10
$\bar{X} \rightarrow$	7.4	9.2	6.4

$$\bar{X} = 7.66$$

Mean of
population = 7.78

- In this example we have taken three samples out of the population each with five observations in it. We computed a Mean for each sample. Note the Means are not the same!
- What would happen if we kept taking more samples?

Standard Error of Mean

The Standard Deviation of the sample Mean distribution, also known as the *Standard Error of the Mean* or just '*Standard Error*'.

Standard Error of the Mean (SEM)

$$S_{\bar{x}} = \frac{S_x}{\sqrt{n}}$$

where :

$S_{\bar{x}}$ = Standard error

S_x = Standard deviation
of the individuals

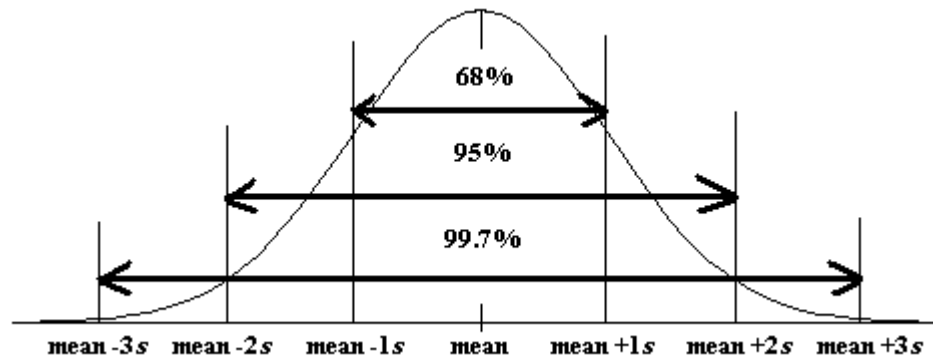
n = Sample size

$$\mathbf{SEM} = \frac{S_x}{\sqrt{n}}$$

Large Sample Sizes – Normal Distribution of Sample Averages

Large Sample Sizes =
Approx Normal Distribution of Sample Averages

Recall that 95% of Normally Distributed data is within approx.
 ± 2 Standard Deviations from the Mean.



Therefore, using the Central Limit Theorem, the probability is 95% that the sample Mean is within ± 2 standard errors of the true mean of the population.

Practical Example – Small Sample Size

Measuring grams of active ingredient in tablets

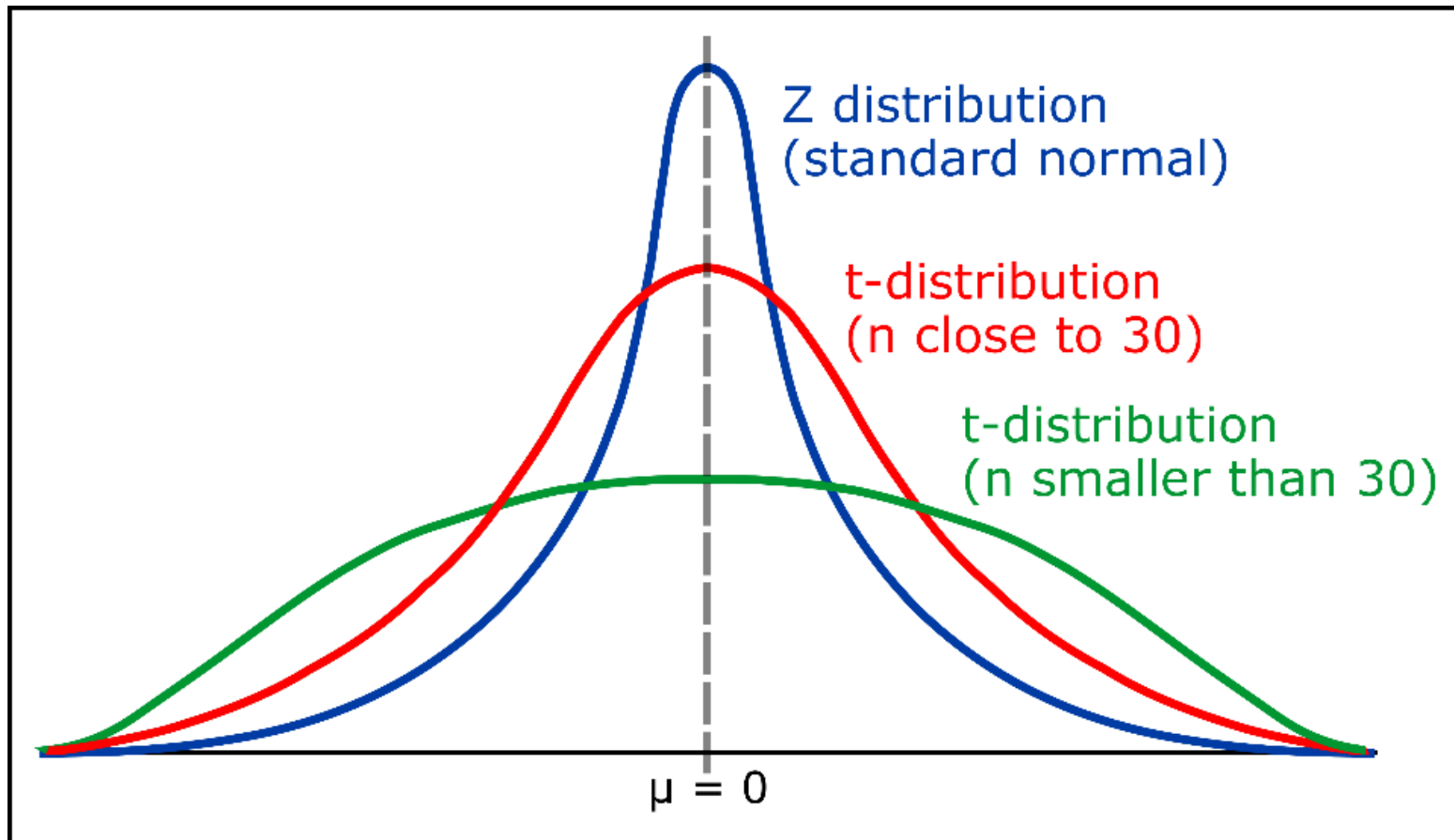
- Based on a sample of 100 tablets you learn your baseline milli-grams of active ingredient is 250 mg with a Standard Deviation of 10 mg.
- Because this is just a sample the 250 mg average is an estimate of the true average.
- Calculate the **95% CONFIDENCE INTERVAL CI**

$$\text{95\% CI : } \bar{X} \pm 2 \frac{S_x}{\sqrt{n}} = 250 \text{ mg} \pm 2 \frac{10}{\sqrt{40}} = 250 \text{ mg} \pm 6.32$$

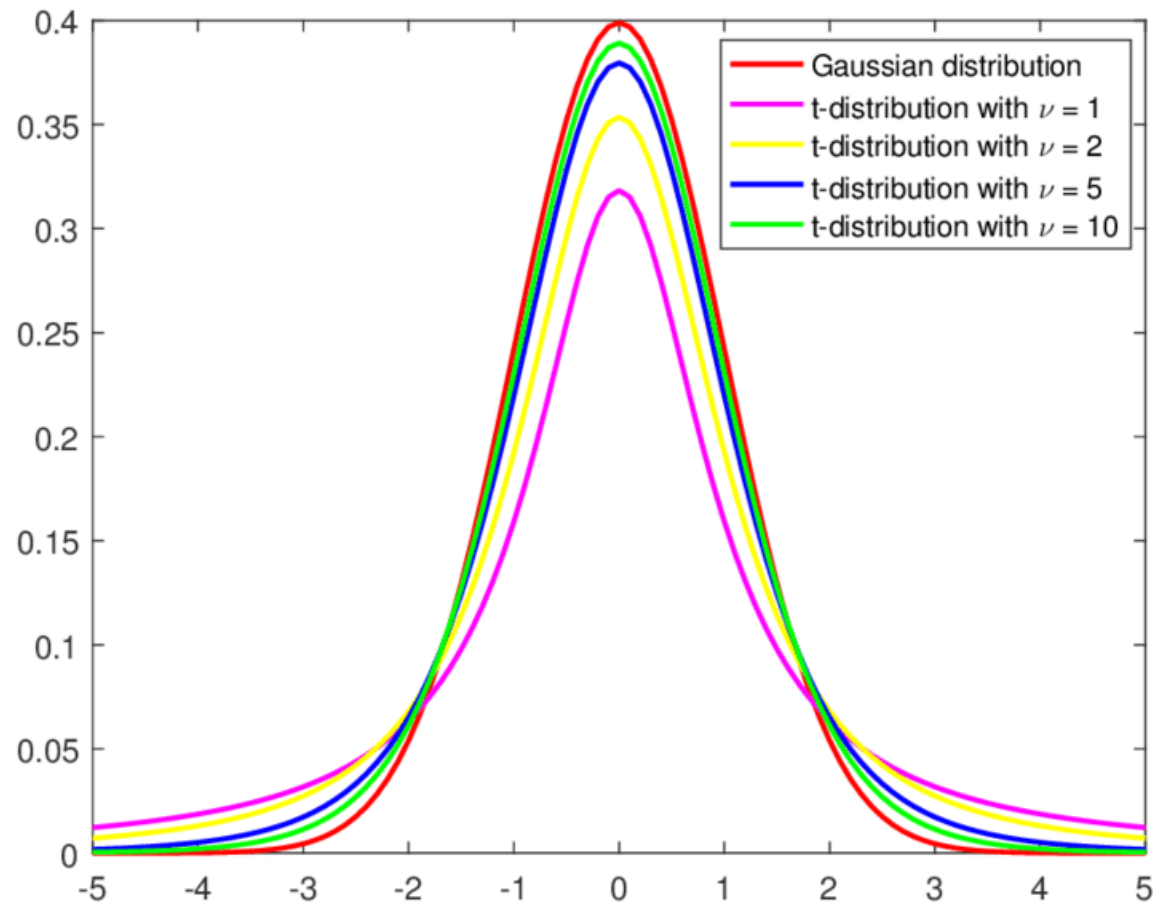
Conclusion:

We can say with 95% confidence that the true average is somewhere between 243.68 and 256.32 mg

Diagram

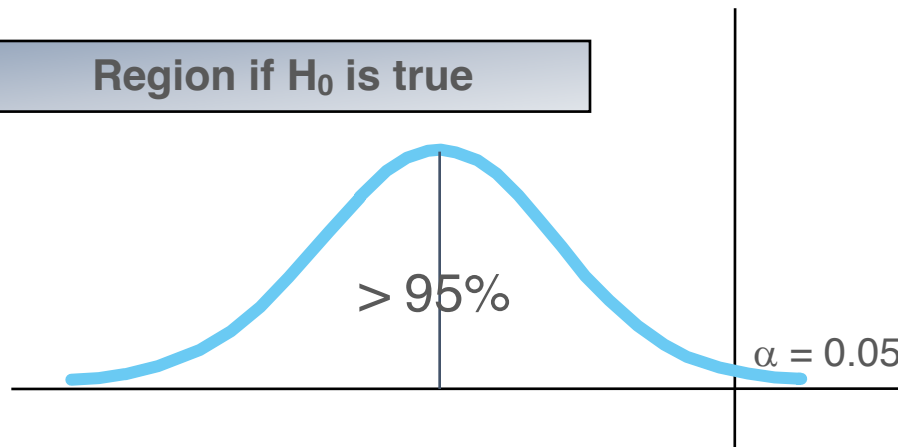


Diagram



One Sided Risk Vs Two Sided Risk

Region if H_0 is true

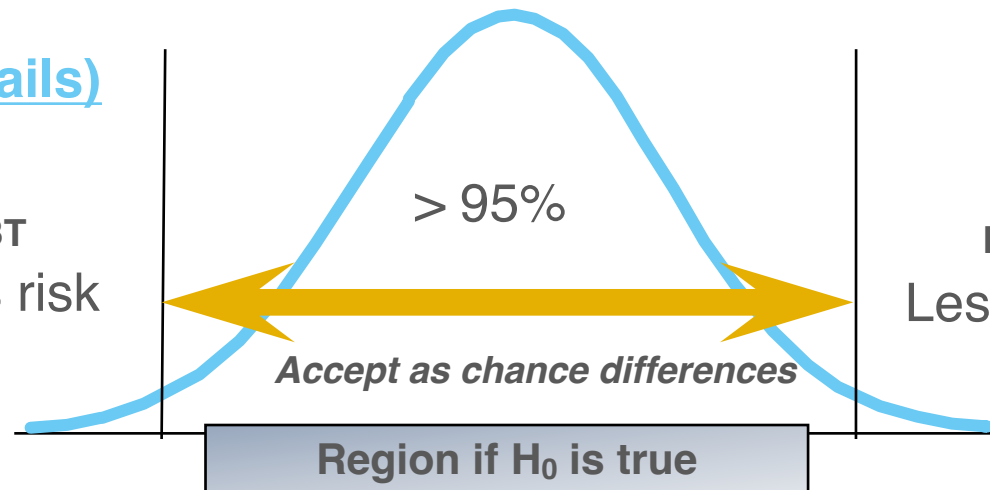


One sided risk (1 tail)

Region of DOUBT
Less than 5% risk

2 sided risk (2 tails)

Region of DOUBT
Less than 2.5% risk



Region of DOUBT
Less than 2.5% risk

Table 1

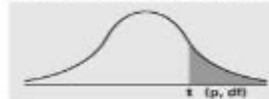
T Table 1

CONFIDENCE LEVEL

	0.600	0.700	0.800	0.900	0.950	0.975	0.990	0.995
DF = n-1								
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756

Table 2

Numbers in each row of the table are values on a t-distribution with (df) degrees of freedom for selected right-tail (greater-than) probabilities (p).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30285	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

Small Sample Sizes : T- Distribution of Sample Averages

Small Sample Sizes =
T- Distribution of Sample Averages

The probability is 95% that the sample Mean is within $\pm t$ standard errors of the true mean of the population.

The value of **t** changes with Sample Size

Practical Example – Small Sample Size

- Based on a sample of **5** tablets you learn your baseline grams of active ingredient is 250 mg with a Standard Deviation of 10 mg.
- Since it is small sample size we use t-distribution
- Degrees of Freedom $n - 1 = 4$
 - - $t = 2.776$

$$\mathbf{95\% \text{ CI : } \quad \bar{X} \pm t \frac{S_x}{\sqrt{n}}}$$

$$= 250 \text{ mg} \pm t \frac{10}{\sqrt{5}} = 250 \text{ mg} \pm \underline{\hspace{2cm}}$$

Conclusion:

We can say with 95% confidence that the true average milligrams of active ingredient is between _____ and _____

Practical Example – Small Sample Size

- Based on a sample of **15** reviews, you learn that your baseline average time taken to review a Credit Card application is **45 minutes** with a Standard Deviation of **10 minutes**.
- Since it is small sample size we use t-distribution
- Degrees of Freedom $n - 1 = 14$
- $t = 2.145$

$$\mathbf{95\% \text{ CI} : } \bar{X} \pm t \frac{S_x}{\sqrt{n}}$$

$$= 45 \pm t \frac{10}{\sqrt{15}} = 45 \text{ minutes} \pm \underline{\hspace{2cm}}$$

Conclusion:

We can say with **95% confidence** that the true average time is between

Summary

At this point you should be able to:

- Explain the term “Inferential Statistics”
- Explain the Central Limit Theorem
- Describe what impact sample size has on your estimates of population parameters
- Understand the difference between a Normal Distribution and t-Distribution
- Explain Standard Error of the Mean (SEM)