



LEAN
SOLUTIONS
ACADEMY

A photograph of three business professionals in an office setting. A woman in a white blouse and black scarf is seated on the left, smiling. Another woman in a light blue shirt is leaning over her from the center, also smiling. A man in a light blue shirt is seated on the right, looking towards the women. The background is a blurred office environment. The image is framed by a dark blue diagonal overlay.

Lean Six Sigma Training **Black Belt**

brought to you by:



**PATRICK
ADAMS**
CONSULTING

INTRODUCTIONS

TIMING and BREAKS

FIRE PROCEDURE

FACILITIES

SPECIAL REQUESTS

The ILSSI Exam

- Done online at ILSSI.org after training is finished
- Login and Password will be provided <https://ilssi.org/login/>
- Proctored by Webcam ([ProctorU.com](https://proctoru.com/))
- Sign-in and Schedule exam on [ProctorU.com](https://proctoru.com/)
- Open Book
- 100 Multiple Choice Questions
- Pass mark is 70%
- Result issued immediately
- Retake is available

ilssi.org



Body of Knowledge

1. **Roles and Responsibilities of Black Belt**
2. **Strategic Planning : Hoshin Kanri**
3. **DMAIC for Black Belts**
4. **Design for Six Sigma DFSS**
5. **Inferential Statistics / Central Limit Theorem / Standard Error**
6. **Advanced Hypothesis Testing (Continuous Data)**
7. **Advanced Hypothesis Testing (Attribute Data)**
8. **Advanced Design of Experiments (DOE)**
9. **Binomial and Poisson Distributions**
10. **Measurement System Analysis for Attribute Data**

Class Schedule

Day 1

- Black Belt Skills and Behavior
- Hoshin Kanri – Strategic Planning
- DMAIC Project Management
- Design for Six Sigma DFSS

Day 2

- Advanced Hypothesis Testing
- Advanced DOE
- Poisson and Binomial

Day 3

- MSA for Attribute Data
- **Black Belt Exam**



The Roles and Responsibilities

OF A BLACK BELT

What is the Role of a Black Belt?

*'There are **four purposes** of improvement:*

- Easier
- Better
- Faster
- Cheaper

These four goals appear in order of priority. Hence the first is to make the work easier for workers while improving the fruits of their labour.'

What is the Philosophy of a Black Belt?

Design and Develop the correct Products and Services

- Build Quality into the Design (DFSS / Quality Planning)

Build Efficient Processes for Delivering the Product

- Constantly improve them
- (Quality Improvement with DMAIC and Kaizen)

Set up Controls to ensure Stability and Quality

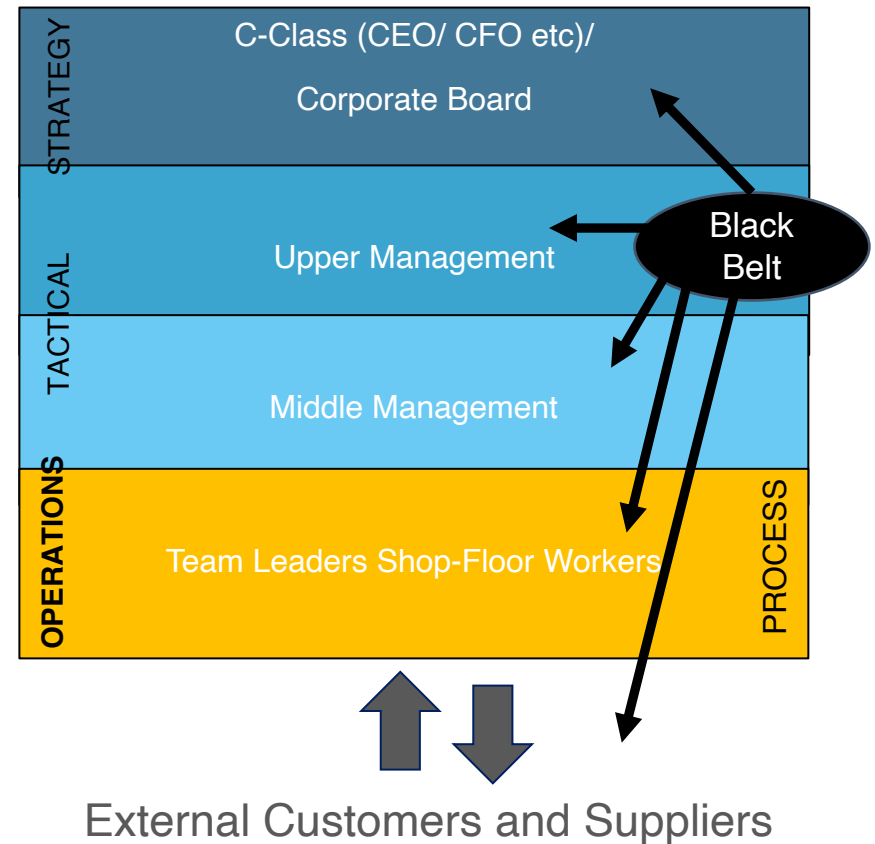
- (Quality Control)

What is the Philosophy of a Black Belt?



The Blackbelt's Place in Business

- Model of an organisation showing where Black Belts are considered in terms of **INFLUENCE** and **DECISION MAKING**.
- The BB should also have a dialogue with **CUSTOMERS** and **SUPPLIERS**

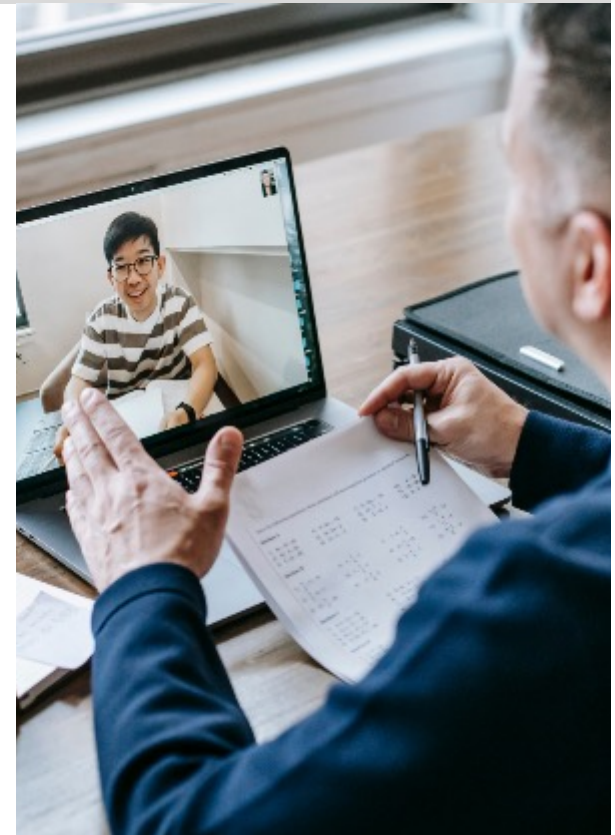


How Blackbelt Leaders Should ‘Think’

- Observe work – “go see” when problems occur and make decisions on facts
- Think and act long term
- Focus on Continuous Improvement
- Challenge management practices and learn new things
- Be humble with little interest in special privileges or status
- Respect and develop people at every level
- Put customers views first
- Believe that lean management is there to improve the entire business
- Soft skills and hard skills are connected and should work together

Blackbelt Skillset

- Leadership skills
- Problem Solving Skills
- Strategic Thinking
- Mentoring, Coaching & Training
- Project Management skills for large projects / programmes
- Practical understanding in use of Statistical tools
- Stakeholder Management
- Risk Management
- Change Management



Blackbelt Consults

Task #1: The Pitch

The Chief Operating Officer (COO) of an investment bank wants to cut the costs in the process for lending high value amounts to new clients. They want to use Lean Six Sigma to achieve the cost cutting targets and asks for your advice.

Give:

1. Two reasons why using Lean Six Sigma could be an advantage.
2. Two challenges that lie in using Lean Six Sigma to achieve the objective (reasons why the initiative might fail).
3. What are some of the first steps to take to implement a Lean Six Sigma programme ?



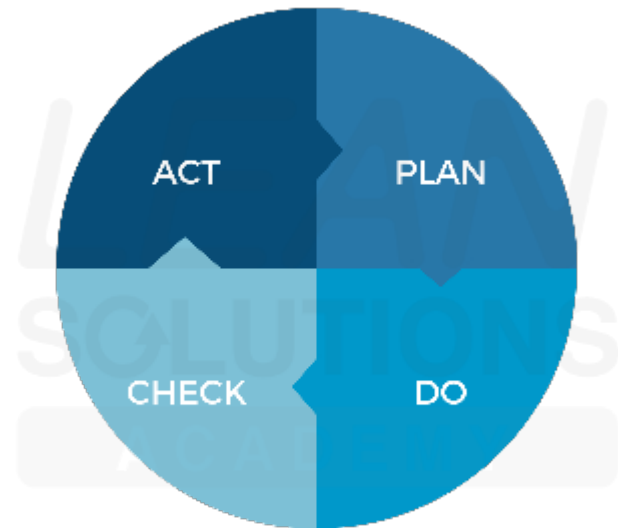
Strategic

PLANNING & DEPLOYMENT

Brief History

Hoshin Kanri was developed in Japan during the 1960s from Quality Management practices in:

- Bridgestone Tire company,
- Toyota,
- Denso,
- Komatsu
- Matsushita (Panasonic).



It was strongly influenced by the [Plan-Do-Check-Act \(PDCA cycle\)](#) of W. Edwards Deming, and Management by Objectives of Peter Drucker.

Brief History

Hoshin = “The Compass” or “Direction Setting”



Kanri = Control with a **Logical Plan** (“The Map”)



“A statement of desired outcome for the future, plus the means of accomplishing that outcome, and for measuring the accomplishment.”

Strategic Principles



CONTINUOUS
IMPROVEMENT

The Hard Stuff

Belted Training / Measure
Projects etc...



RESPECT &
DEVELOPMENT
OF PEOPLE

The Soft Stuff

Behaviours, Motivation
How People Think

Strategic Principles

FAKE LEAN



CONTINUOUS
IMPROVEMENT

We Focus on Just CI

SUSTAINABLE LEAN



CONTINUOUS
IMPROVEMENT



RESPECT &
DEVELOPMENT
OF PEOPLE

We Put More Focus on Respect
and Development of People

Strategic Long-Term Objectives

Process
Improvement
& Innovation

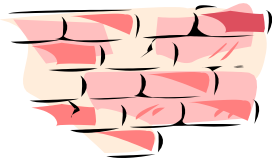
Value Creation
for Customers

Workplace
Balance
& Harmony

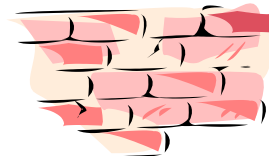
Financial
Sustainable
Improvement

Strategic Principles

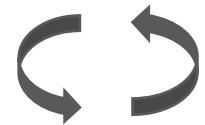
Starting Point



Fake Lean



True Lean



Added Value – Lean Behaviors



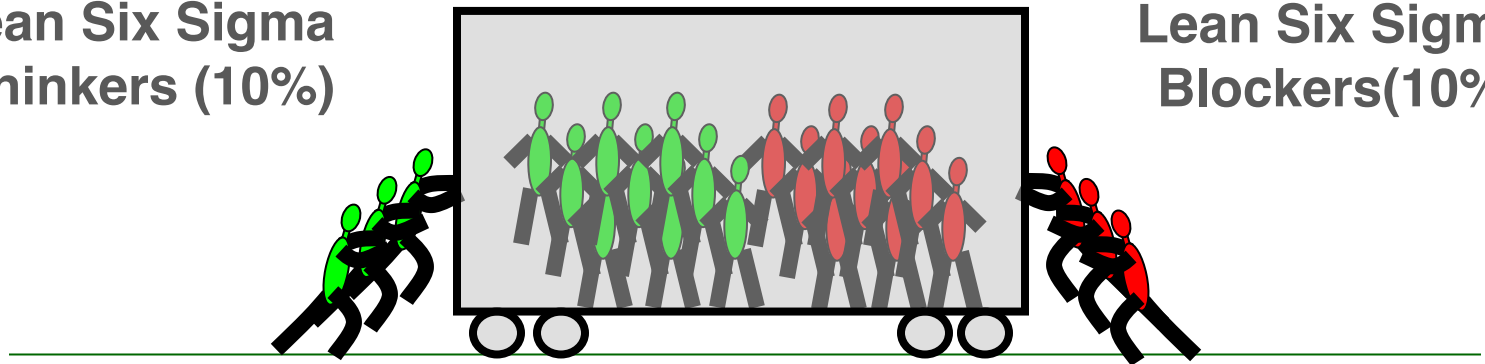
A Typical Workforce...

Willing to
Support (40%)

**Lean Six Sigma
Thinkers (10%)**

Quick to
Criticize (40%)

**Lean Six Sigma
Blockers (10%)**



The direction you want to go

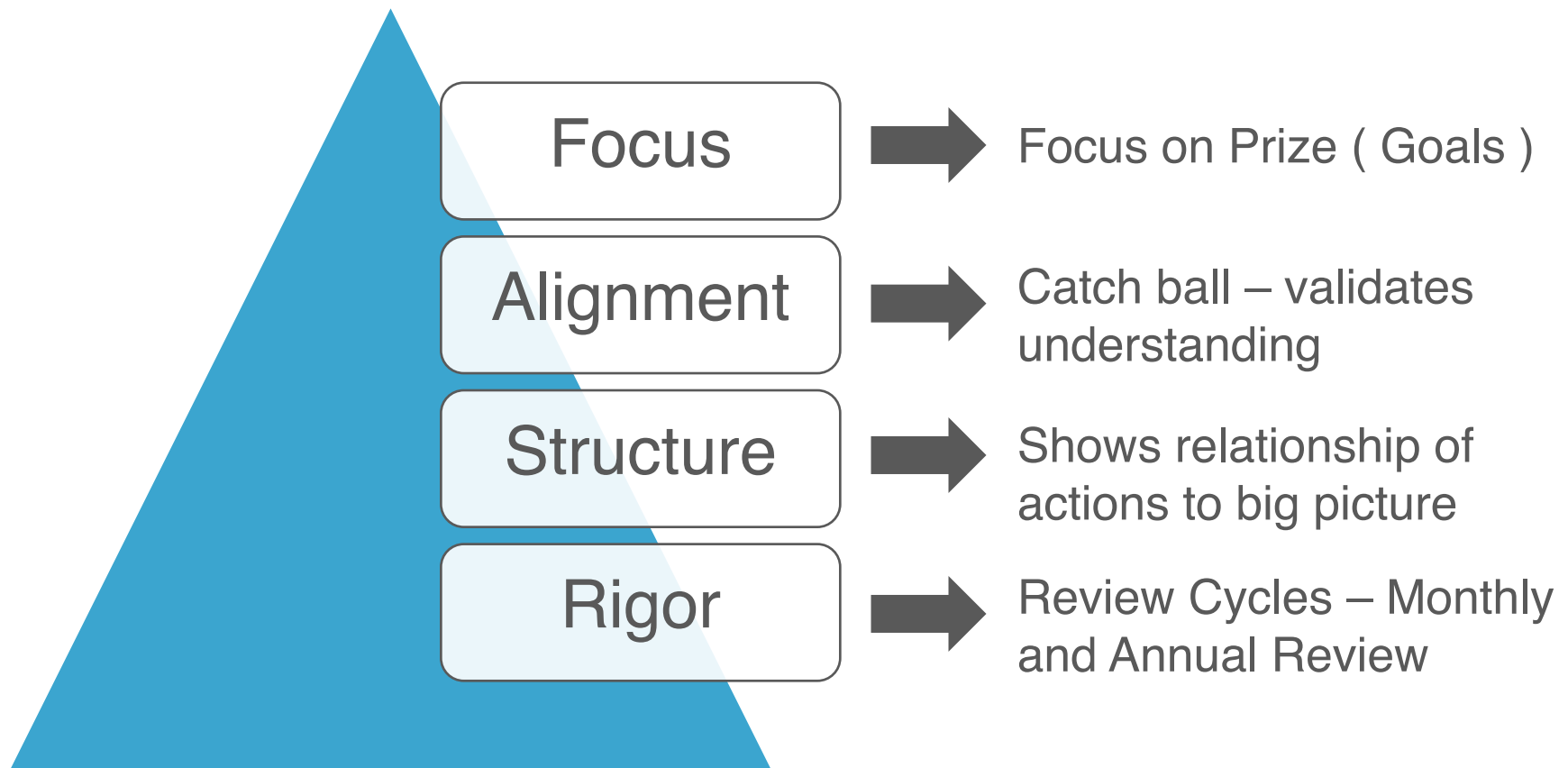
Keep on Pushing

What is Organizational Culture?

“ The way we do things around here”

- The beliefs, attitudes and values that exist within your organisation.
- Every company will have factors that influence its company culture.

Four Elements of Hoshin Kanri



Hoshin Planning Process

C
A
T
C
H
B
A
L
L

PLANNING



1. Establish Vision / Mission for Business
2. Develop 3 to 5 Year Strategic Objectives
3. Develop Annual Objectives and Improvement projects
4. Prioritize and Schedule Improvement Projects

IMPLEMENTATION

Lean SS
Projects

5. Implement Improvement projects

REVIEW



6. Monthly Review
7. Annual Review

Vision / Mission / Values

Vision

- An aspirational statement illustrating how the organisation will look in the future and how it will benefit if the change happens

Mission

- A statement defining what an organization is, why it exists, its reason for being.

Values

- Fundamental beliefs of the organization. These guiding principles dictate behavior and help employees understand the difference between right and wrong.

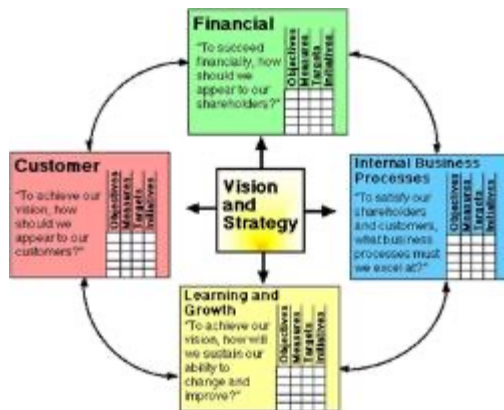


“A Black Belt must be committed to the company Vision, Mission and Values”

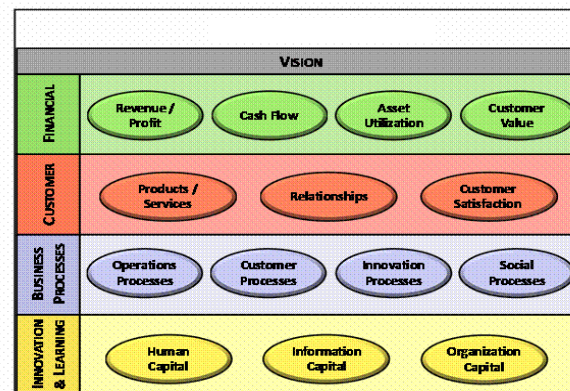
Develop Strategic Objectives

Strategic Planning is determining those 3 to 5 year breakthrough objectives that will move the organization towards world class.

Balanced Score Card



Strategy Maps



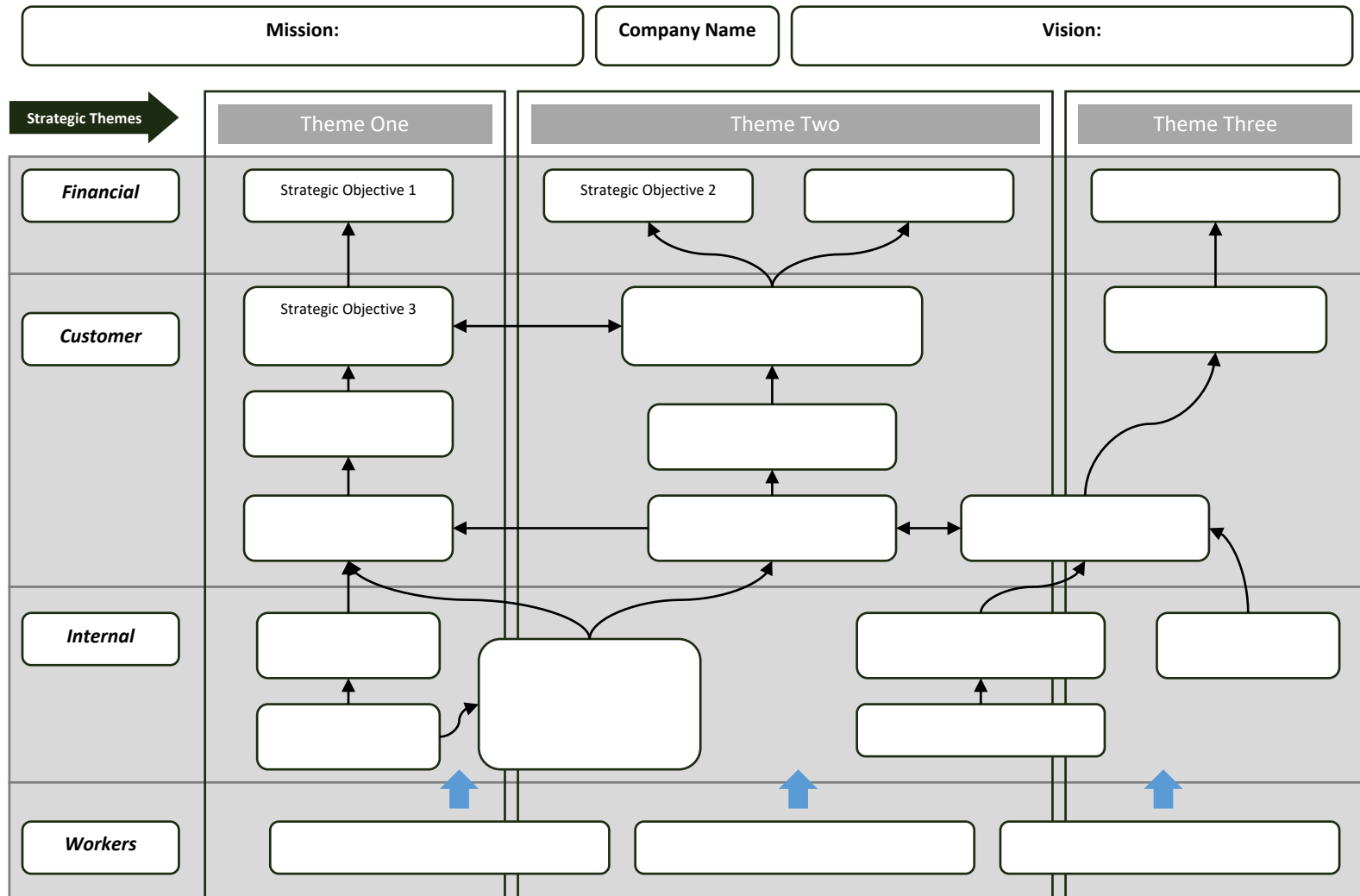
SWOT Analysis



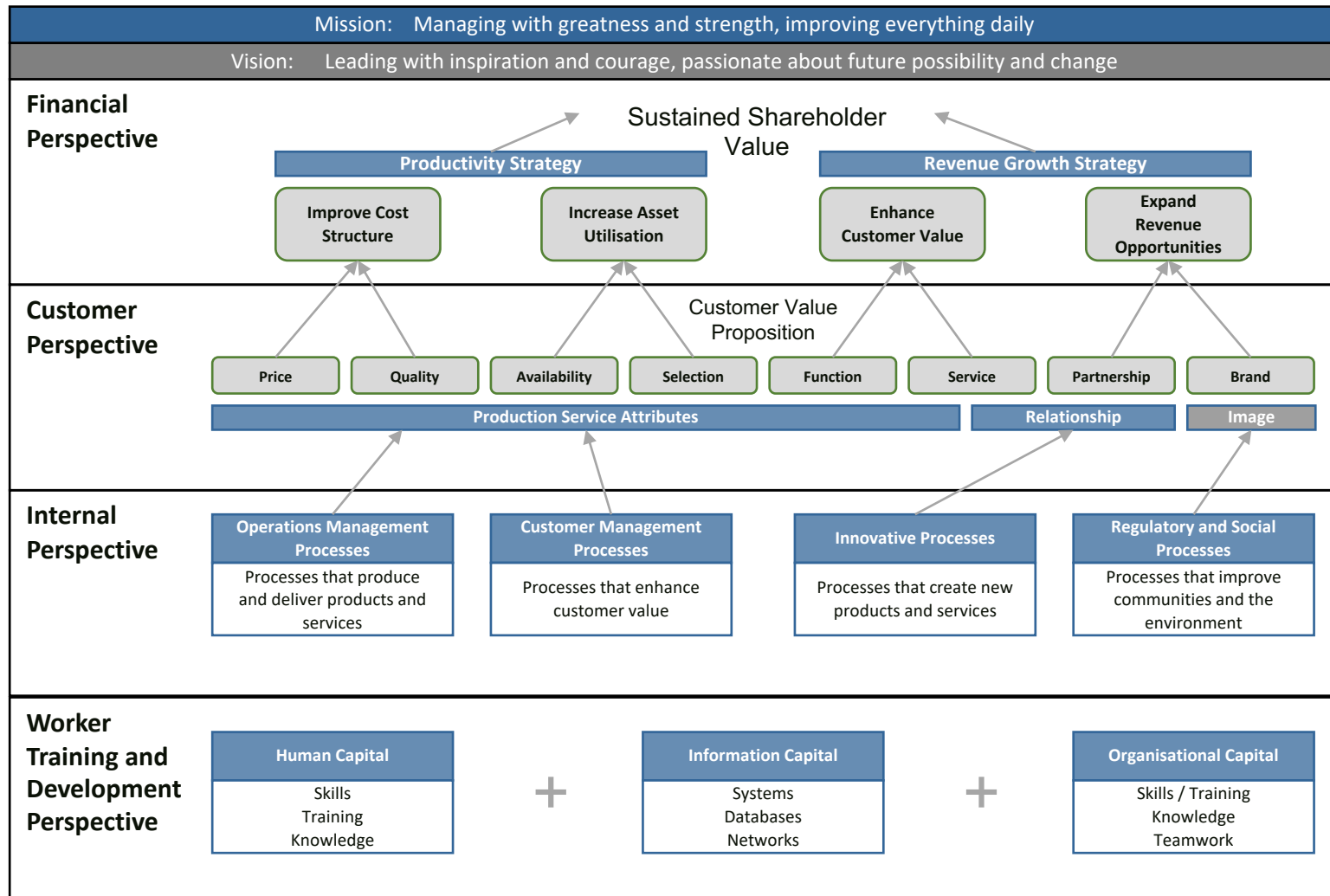
Basic Strategy Map

Financial	
Customer	
Processes	
Workers	

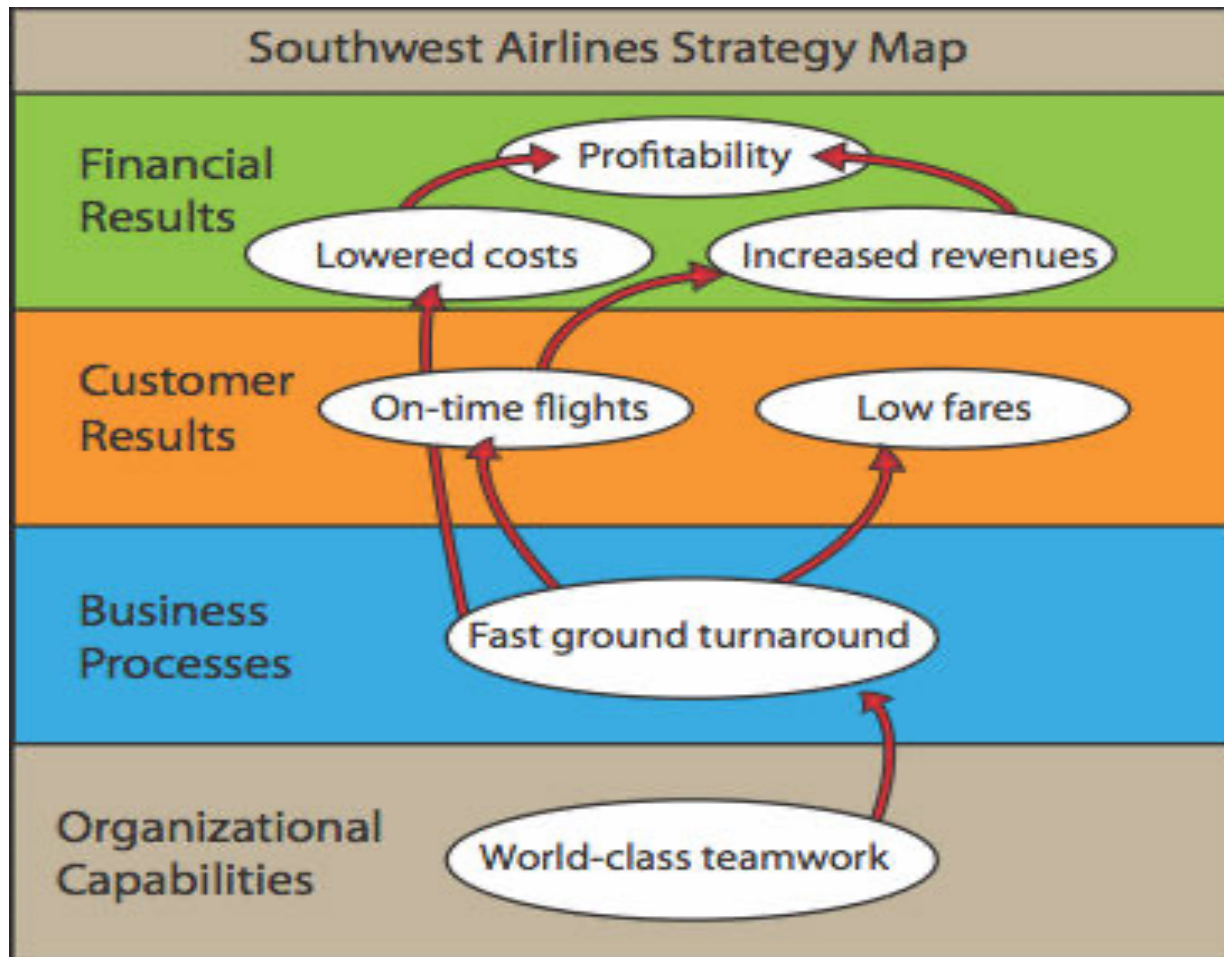
Add More Detail



Strategy Man – Generic Example



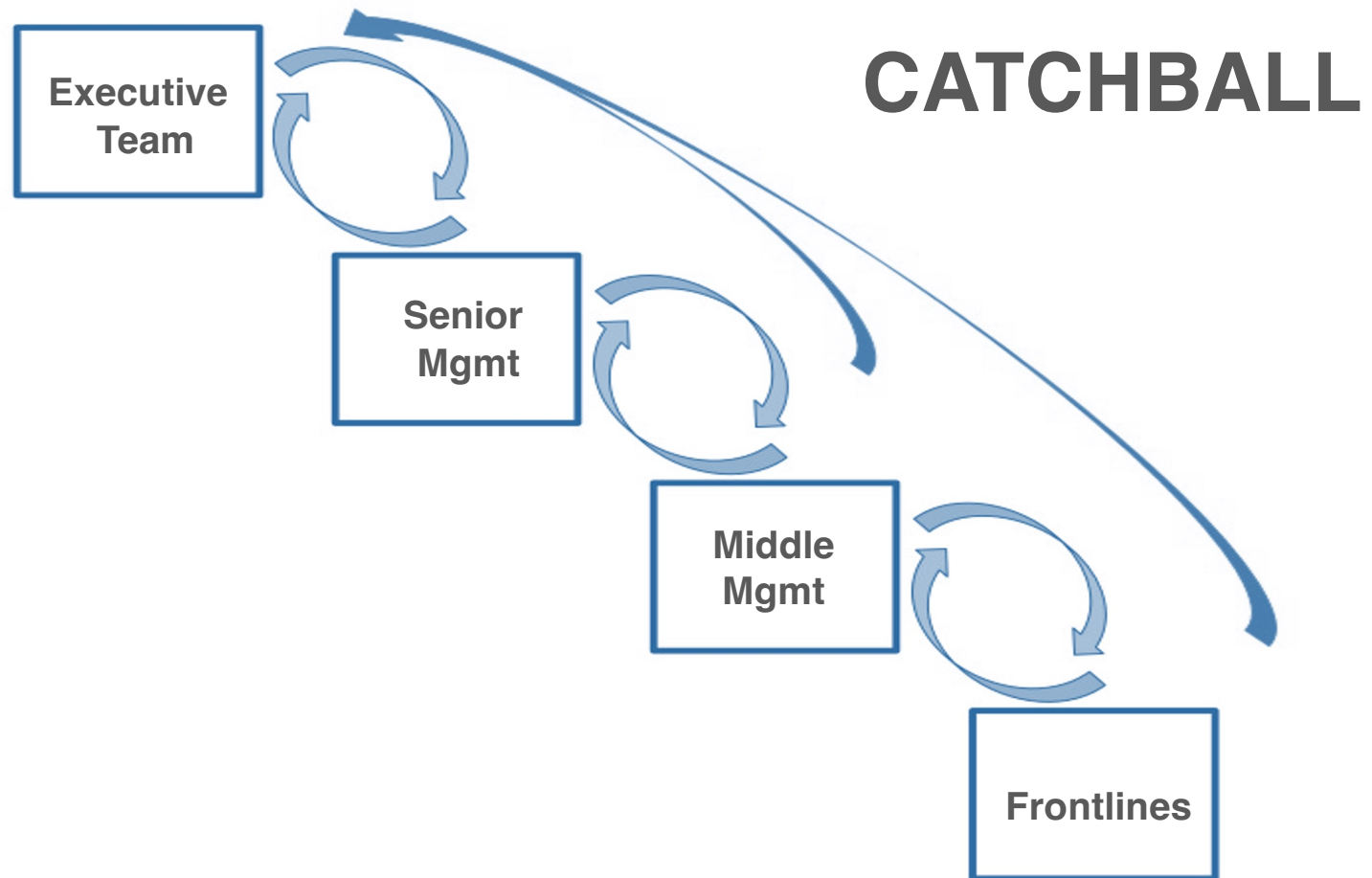
Strategy Man – Specific Example



Catch – Ball Process



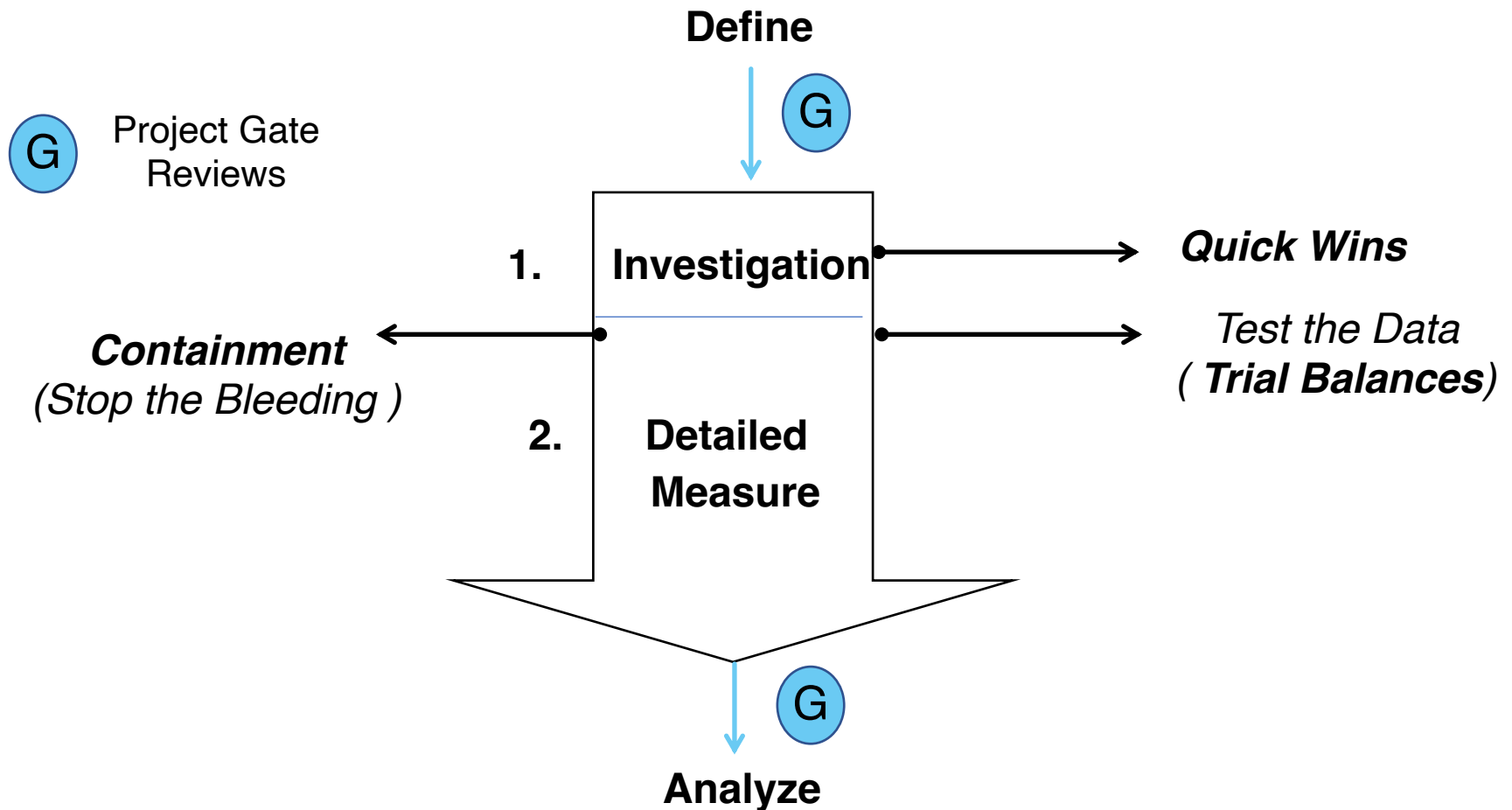
Strategy Deployment



DMAIC Projects

FOR BLACK BELTS

Two Step Measure



Trial Balances, Quick Wins, & Containment Actions

Trial Balances

- Try analysing your data early on (days/weeks) – you will not have enough data to identify root causes but you will be able to see if anything is missing

Quick Wins

- Just by measuring something you often suddenly see something obvious to fix – this might reduce the problem or give another benefit.
JUST DO IT THERE AND THEN !

Containment

- A 'Work Around' to prevent further problems
- A 'Cause' of the problem is identified and a 'fix' put in place
- Often an interim solution.
- 'Stopping the Bleeding'

Project Portfolio Management

Project portfolio management (PPM) is a key skill of any Black Belt with accountability for multiple projects. (also called '*Programme Management*')



Control



Efficiency



Visibility



Governance

Efficiency and Control

Use a Project Portfolio Management (PPM) tool for better
Efficiency and Control

Companion by Minitab

<http://www.minitab.com/en-us/products/companion>

COMPASS

<https://www.creato.com/compass-qms>

Microsoft Project Server 2016 / Project Portfolio Management (PPM)

<https://products.office.com/en-gb/project/enterprise-project-server>

Managing Stakeholder Expectations

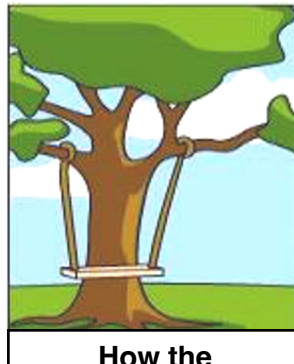
Black Belt Governance duties at the programme level:

- Selection of the correct Projects
- Alignment of Projects with Company Vision (Hoshin Kanri)
- Gate Reviews
- Status Reports to Upper Management / Executives
- Risk Management of portfolio / programme
- Change Management for all projects

Managing Stakeholder Expectations



**How the customer
explained it**



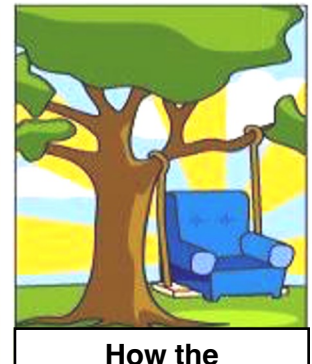
**How the
project leader
understood it**



**How the analyst
designed it**



**How the
programmer
wrote it**



**How the
consultant
described it**



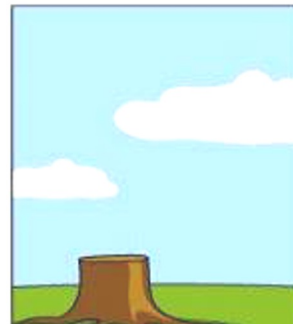
**How the
project was
documented**



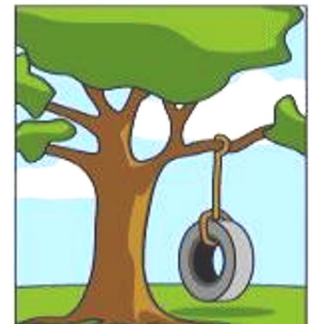
**What was
installed by
operations**



**How the customer
was billed**



**How the project
was supported**



**What the
customer really
needed**

Change Management

- **Requests for Change** (RFCs) are written proposals for change to any existing process
- The BB needs to make sure that there is a solid Change Management process for all Lean Six Sigma projects
- A **Change Control Board** (CCB) is often used as part of the Change Management Process for Approval / Rejection of RFCs
 - For more information of RFC, CCB and Change Management see:
 - <https://alison.com/courses/introduction-to-change-management/content>



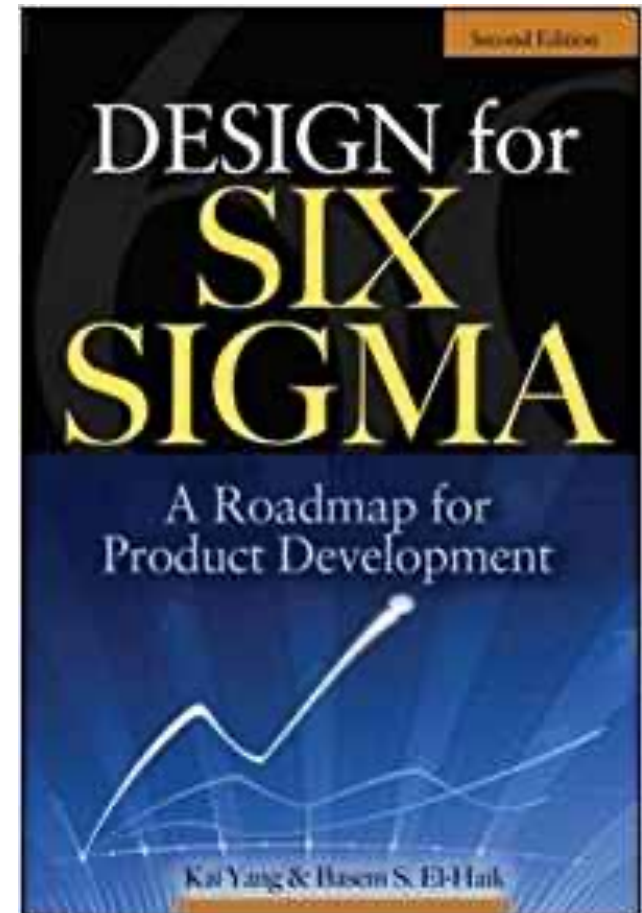
Design For

Six Sigma DFSS

Literature

Design for Six Sigma: A Roadmap for Product Development

2008 - By [Kai Yang](#) [Basem Ei-Haik](#)



Why Use Design for Six Sigma?

Design For Lean Six Sigma because it...

- Reduces the number of **DEFECTS** delivered to our customers
- Reduces the **REWORK and SCRAP** in the production of our products, processes or services
- Reduces the **LEAD TIME** in the delivery of our products, processes or services
- Reduces **COST** of production and delivery of our products, processes or services

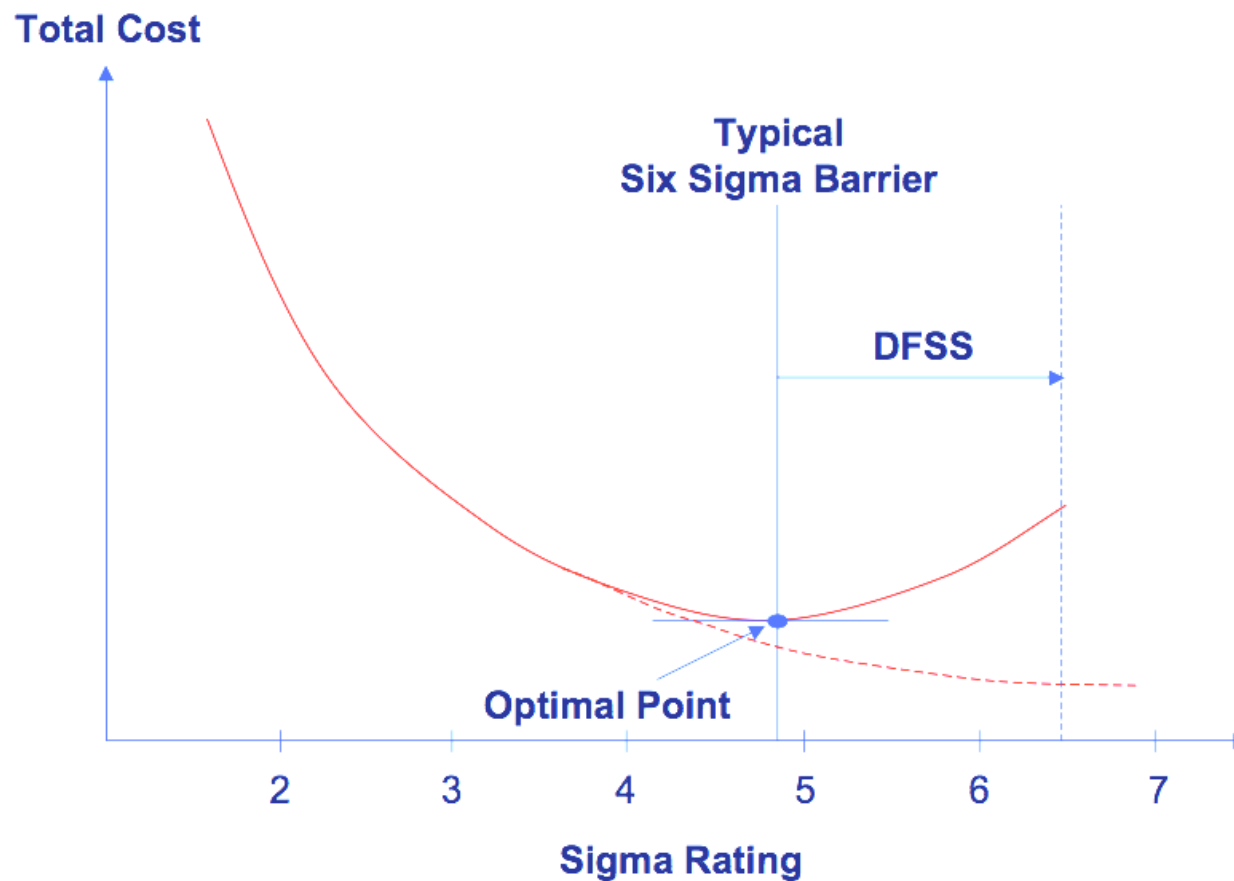
DFLSS to Reduce the Risk of New Product Launches

- Launching new products, processes and services has historically been a high risk activity.
- More than 85 % of new Fast Moving Consumer Goods (FMCG) products that established companies put into the market fail to deliver the expected return on investment.

(Neilson Group 2014)

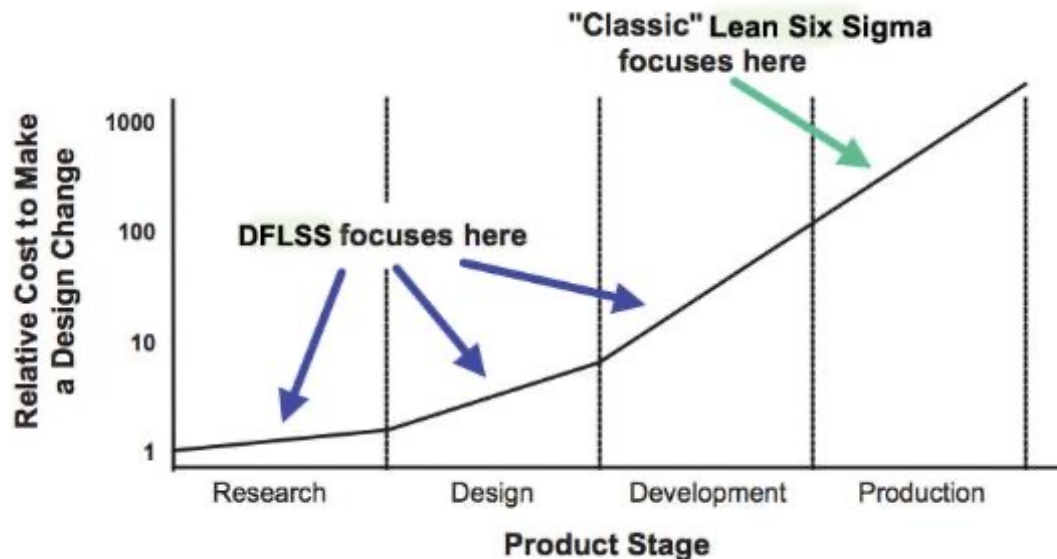
www.nielsen.com/us/en/insights/news/2014/how-to-flip-85-misses-to85-hits-lessons-from-the-nielsen-breakthrough-innovation-project.html

DFLSS to Break the 5 Sigma Barrier



When to Use DFLSS

DFSS is always used EARLY in the new product development lifecycle.

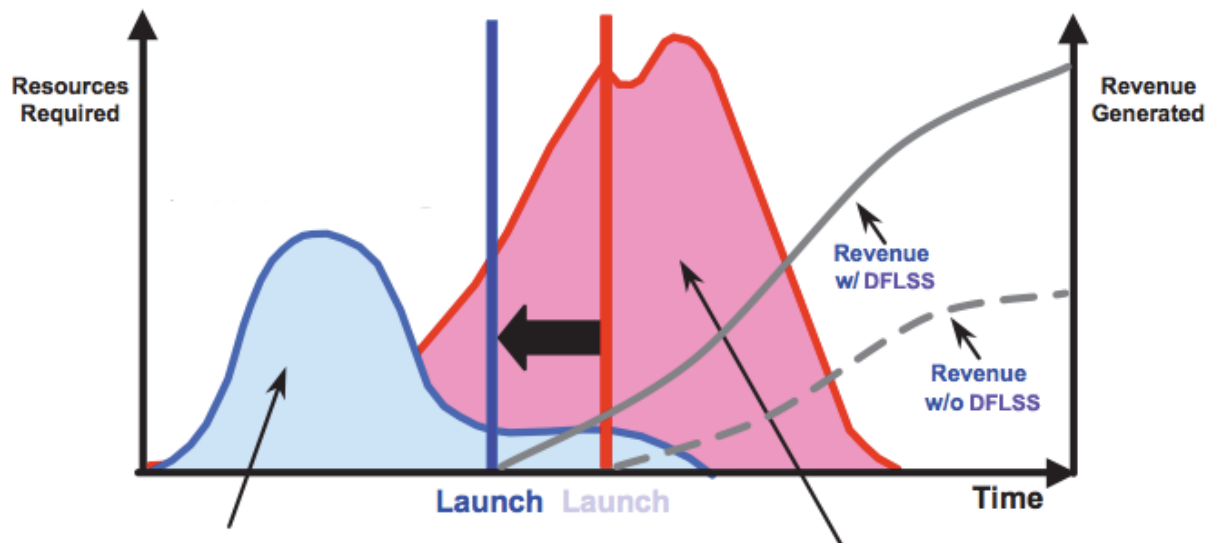


Why?

Because Cost of Changes increases with time / stage in product lifecycle.

DFLSS: Predictive vs Reactive Design

Front-End Loading of Resources for Predictive Design:



With DFLSS : Predictive Design

- Early problem identification; solution when costs low
- Faster market entry: earlier revenue stream, longer patent coverage
- Lower total development cost
- Robust product at market entry: delighted customers

Without DFLSS : Reactive Design

- Unhappy customers
- Unplanned resource drain
- Skyrocketing costs
- Next product compromised

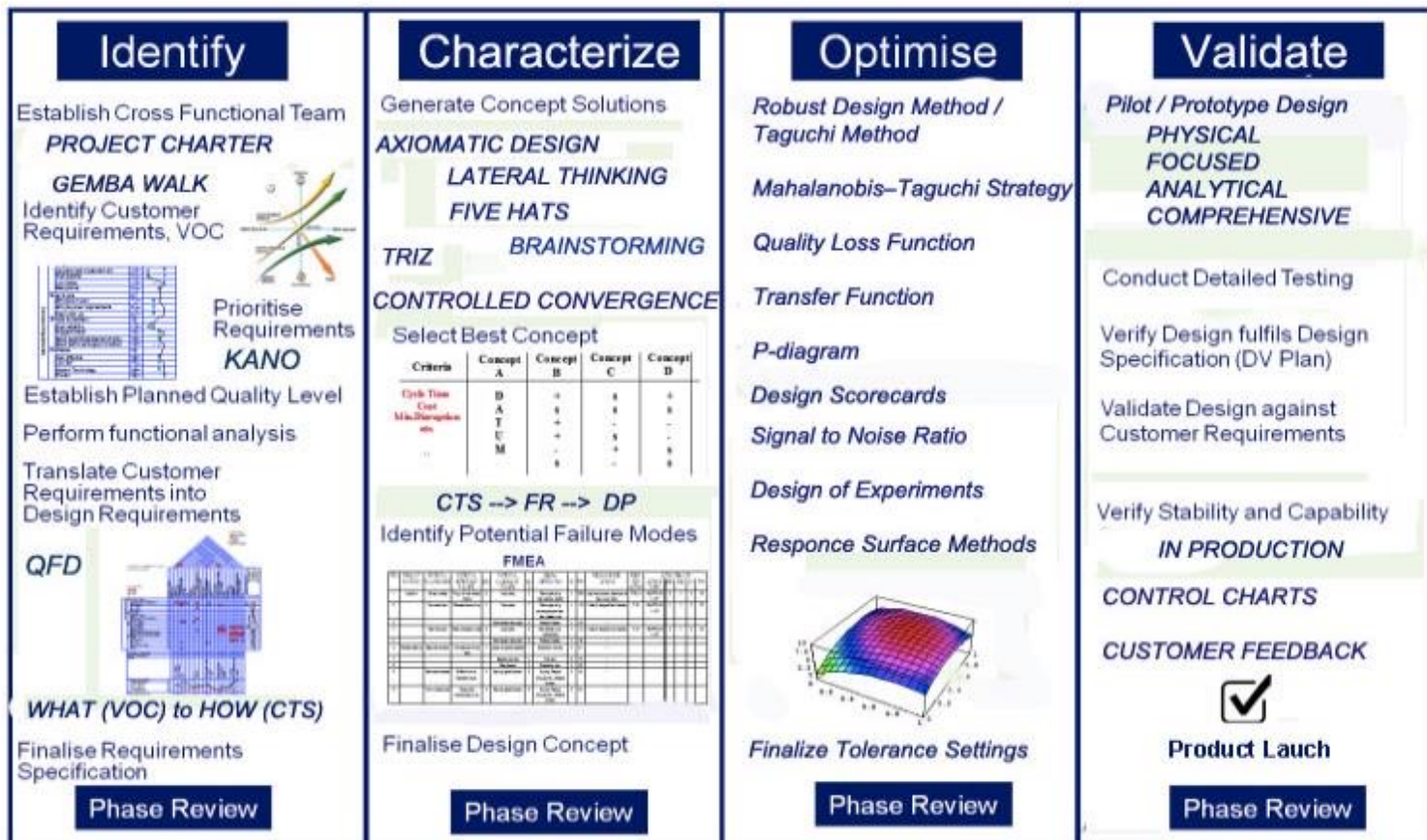
DFLSS Deployment Methodologies (Design Algorithms)

Several Popular Methodologies (Design Algorithms):

- **DMADV** (Define, Measure, Analyse, Design, Validate).
- **ICOV** (Identify, Characterize, Optimize, Validate)
- **IDOV** (Identify, Design, Optimize, Validate).
- **I2DOV** (Invent / Innovate, Develop, Optimize, Validate).
- **CDOV** (Concept, Design, Optimize, Validate).

*Note: All approaches use the same philosophical approach.
It is only the vocabulary that is used that is different.*

ICOV Methodology



Steps of Methodology - Roadmap

Step 1

Identify

- **FORM TEAM**
- **DEFINE / IDENTIFY / UNDERSTAND** your customer, Understand your market, Understand their needs and wants. Understand what **FUNCTIONAL REQUIREMENTS**.
- **(FR)** are important to them. Understand the competition. Understand the environment. Survey and Measure to confirm all understanding.

Step 2

Characterize

- **INNOVATE / CONCEPTUALIZE / CHARACTERIZE** a new or improved product or service which will satisfy FR to satisfaction of customers. **SELECT** best option.
- **DESIGN and DEVELOP** the concept into a working model.

Step 3

Optimize

- **OPTIMIZE** the new / improved model by experiment, measurement and analysis. Adjust tolerances to make the model as robust and free of potential defects as possible.

Step 4

Validate

- Prototype / Pilot the optimized working model to **VALIDATE / VERIFY** that it meets the demands of the customer. Test and measure capability of the new/improved design to meet the expectations of customers. Release into production and continue to monitor.

Quality Function Deployment - QFD

The House of Quality

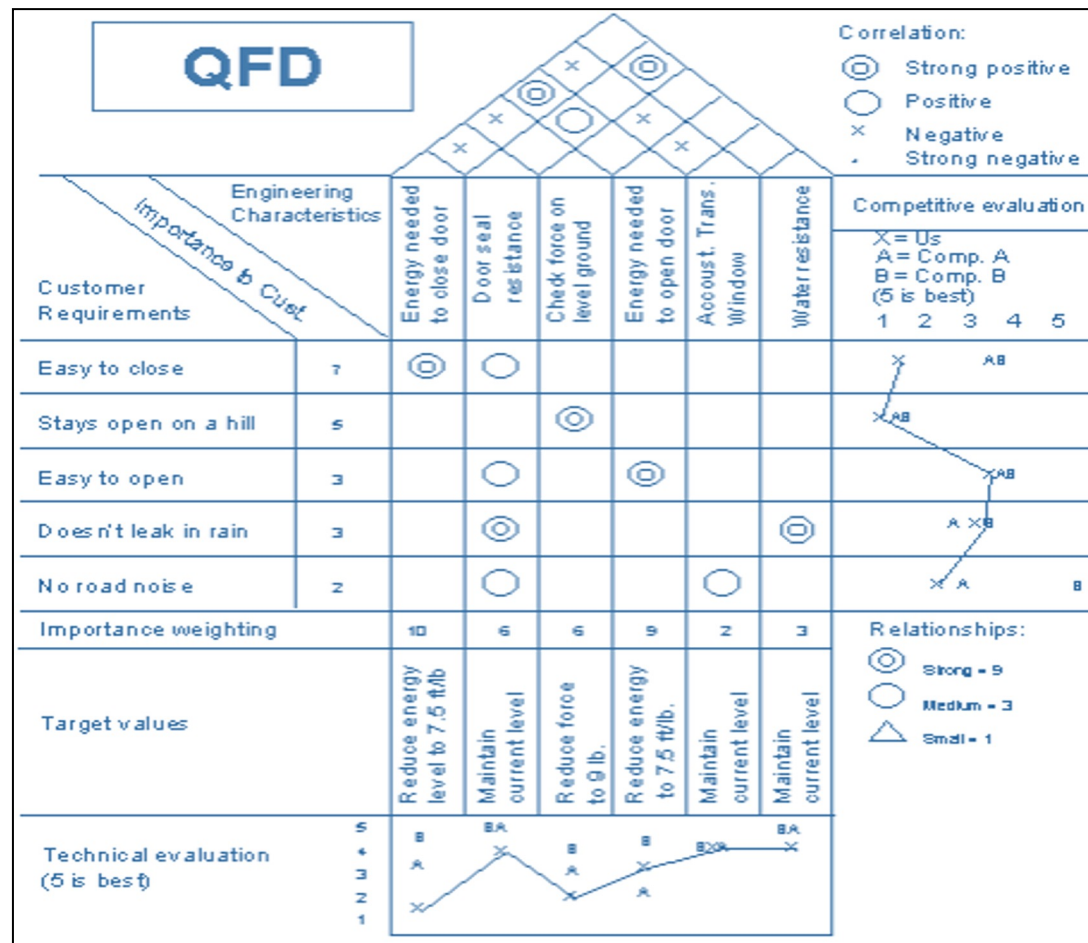


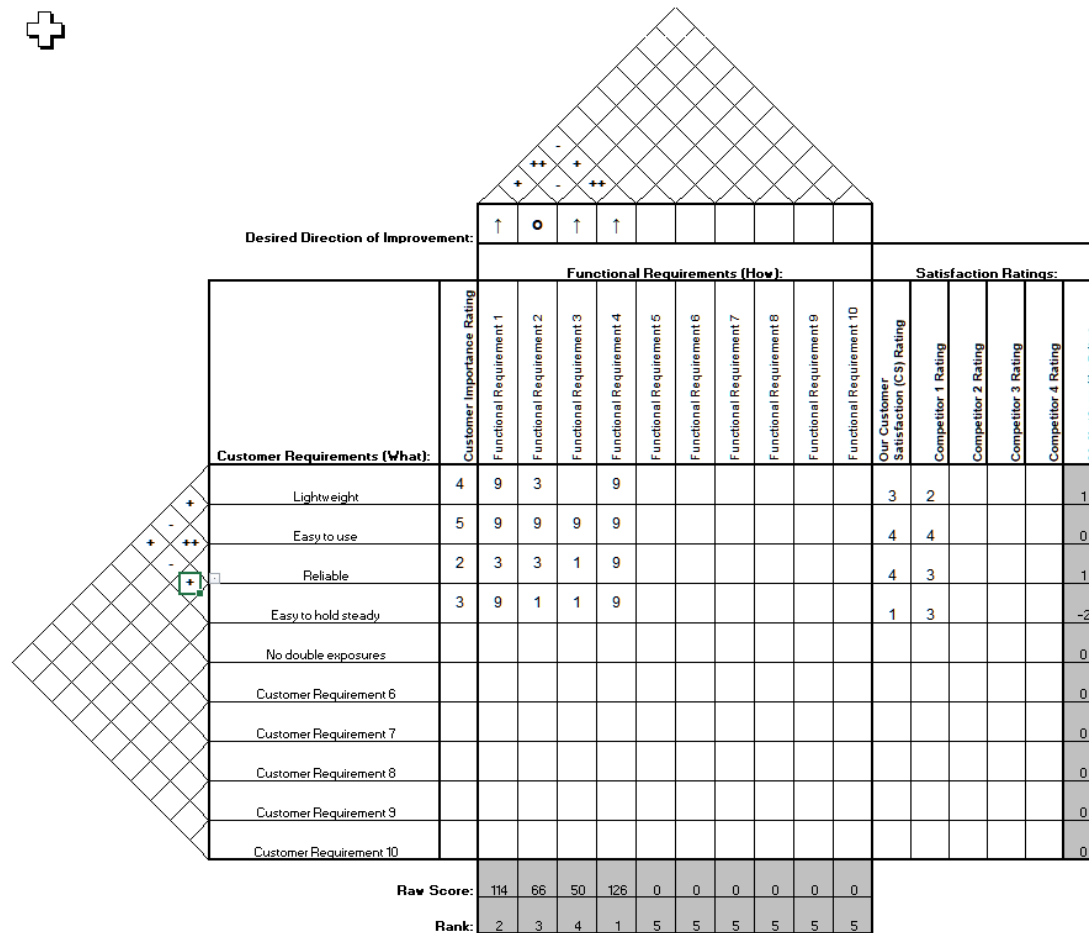
Quality Function Deployment Overview

- Quality Function Deployment (QFD) is a structured approach to understanding true customer priorities and a business's ability to fulfil them
- QFD starts by identifying customer needs, wants, and wishes then translates them into the features and functionality of the product or service to meet those needs
- It can be used when-ever you are designing or developing a new process where customer satisfaction is key
- Typically uses a four step matrix based approach (House of Quality)



House of Quality



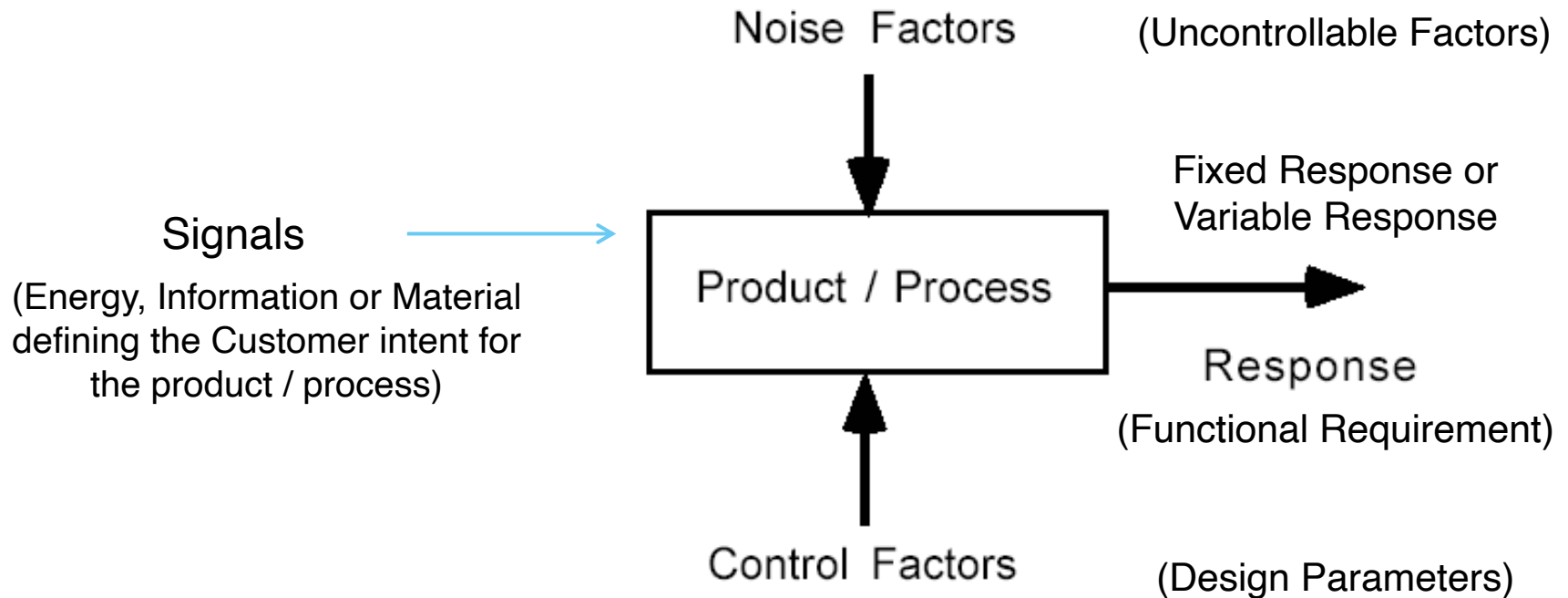


P-Diagrams

P-Diagram (Parameter Diagram of Process Diagram)

- A graphical tool to help visualize the design problem in DFSS methodology.
- **Provides a structured method to identify:**
 - Intended Customer Inputs (Signals) M
 - Unintended Inputs (Noise Factors) N
 - Design Controllable Factors X
 - Intended Outputs, Response (Ideal Function) Y

P-Diagrams



At least 1 Control Factor (DP) per Functional Requirement (FR)

Example - Refrigerator

- CTQ = Store fresh food in the home for both long term and short term use
- FR1 = Freeze food for long-term preservation
- FR2 = Maintain food at cold temperature for short term preservation

To satisfy these two FRs, a refrigerator with two compartments is designed.

Two DPs for this refrigerator may be stated as:

- DP1 = The freezer section
- DP2 = The chiller (refrigerator) section



Example of Decompose - Refrigerator

Having chosen the DP1, we can now decompose FR1 into secondary functional requirements as: **FR1**

FR11 = Control temperature of the freezer section in the range of -18 C +/- 2 C

FR12 = Maintain the uniform temperature throughout the freezer section at the preset temperature

FR13 = Control humidity of the freezer section to relative humidity of 50 %

Class Exercises

1. Fizzy Drink Can

- Name the Primary FRs for a Fizzy Drinks can
- Name the Primary DPs for a Fizzy Drinks can



2. A Umbrella

- Name the Primary FRs for a Umbrella
- Name the Primary DP's for a Umbrella



3. A Running Shoe

- Name the Primary FRs for a running shoe
- Name the Primary DPs for a running shoe



Robustness Design

Robustness means insensitivity to variations caused by noise factors, which may include environmental factors, user conditions, and manufacturing disturbances.

<https://www.youtube.com/watch?v=DiLD6ACw7fQ>

Shin Taguchi (son of Genichi Taguchi)

Cost vs Quality Trade Off

Taguchi Loss Function

Other Names Include:

- Taguchi Quadratic Loss Function
- Quality Loss Function

The Taguchi Loss Function translates variation and ‘Off Target’ production into a monetary value \$\$\$ lost in customer value.

Taguchi Loss Function

$$L = k * E (Y - T) ^ 2$$

Where

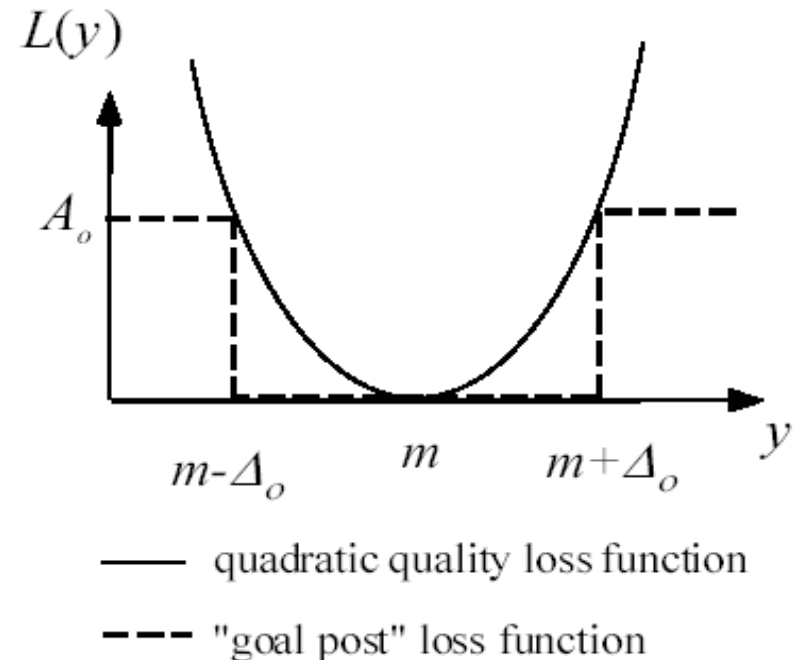
- L** = value loss (in \$\$)
- k** = quality loss coefficient
- E** = Expected value by customer
- T** = Target value by supplier
- Y** = Actual value

The objective of Taguchi Tolerance Design is to minimize L by finding the perfect balance between reduced tolerances and increased costs of production

Quality Loss Function

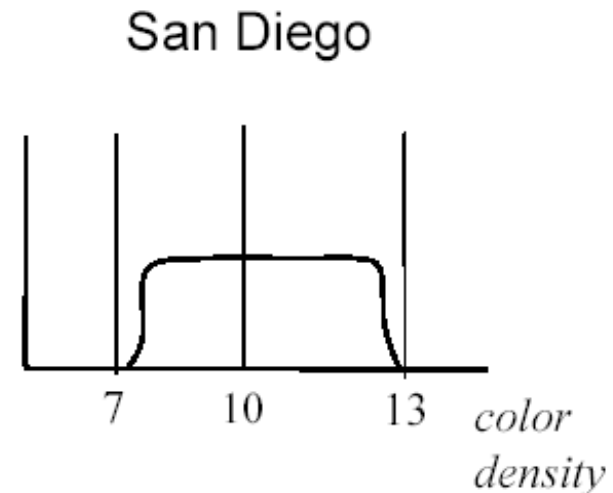
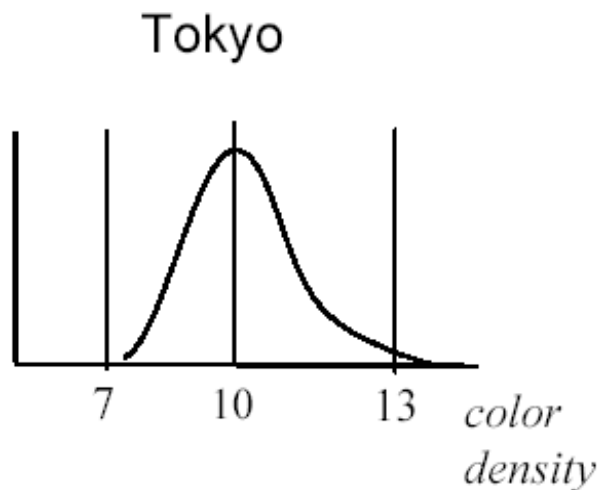
\$\$\$ Loss due to deviation from target quality

- Zero at the target value
- Equal to scrap cost (A) at anywhere outside the tolerance limits



Example – Sony Televisions

- Sony TVs manufactured in both JAPAN and USA
 - See: <https://statweb.stanford.edu/~owen/courses/363/lecture13.pdf>
- Distribution of color density varies around the mean
 - Which has lower defect rates?
 - Which one has better quality?



Inferential Statistics

The Central Limit Theorem

Standard Error of the Mean

Inferential Statistics Defined

Inference :

*A conclusion reached on the basis of evidence and reasoning.
(Oxford Dictionary)*

There are 2 goals in using Inferential Statistics :

1. To draw a conclusion about a population based on the analysis of sample data from the population.
2. To quantify the level of confidence that we have in our conclusion. To understand the level of risk that we are taking in acting upon our conclusion.

The Central Limit Theorem

The Central Limit Theorem says:

As the sample size becomes larger the distribution of means of the samples will form a Normal Distribution, the spread of the distribution becomes narrower, and the mean of the means will move towards the mean of the population.

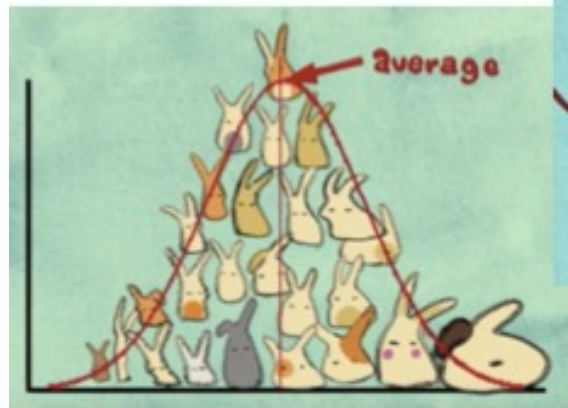
The Central Limit Theorem is key to Inferential Statistics

- The first version of this theorem was postulated by the French-born mathematician Abraham de Moivre who, in a remarkable article published in 1733, used the normal distribution to approximate the distribution of the number of heads resulting from many tosses of a fair coin.

The Central Limit Theorem

Bunnies, Dragons and the 'Normal' World: Central Limit Theorem | The New York Times

<https://www.youtube.com/watch?v=jvoxEYmQHNM>



Sampling Distributions – The Foundation of Statistics

Population

Samples from the population, each with five observations:

3
5
2
12
10
1
6
12
5
6
12
14
3
6
11
9
10
10
12

	<i>Sample 1</i>	<i>Sample 2</i>	<i>Sample 3</i>
	1	9	2
	12	8	3
	9	5	6
	7	14	11
	8	10	10
$\bar{X} \rightarrow$	7.4	9.2	6.4

$$\bar{X} = 7.66$$

Mean of
population = 7.78

- In this example we have taken three samples out of the population each with five observations in it. We computed a Mean for each sample. Note the Means are not the same!
- What would happen if we kept taking more samples?

Standard Error of Mean

The Standard Deviation of the sample Mean distribution, also known as the *Standard Error of the Mean* or just '*Standard Error*'.

Standard Error of the Mean (SEM)

$$S_{\bar{x}} = \frac{S_x}{\sqrt{n}}$$

where :

$S_{\bar{x}}$ = Standard error

S_x = Standard deviation
of the individuals

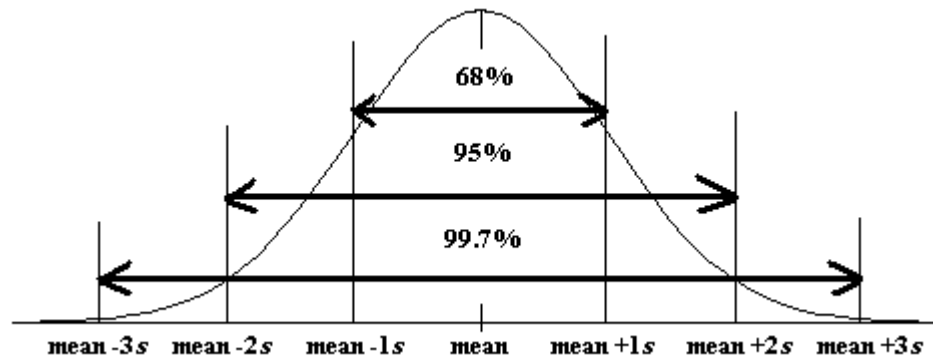
n = Sample size

$$\mathbf{SEM} = \frac{S_x}{\sqrt{n}}$$

Large Sample Sizes – Normal Distribution of Sample Averages

Large Sample Sizes =
Approx Normal Distribution of Sample Averages

Recall that 95% of Normally Distributed data is within approx.
 ± 2 Standard Deviations from the Mean.



Therefore, using the Central Limit Theorem, the probability is 95% that the sample Mean is within ± 2 standard errors of the true mean of the population.

Practical Example – Small Sample Size

Measuring grams of active ingredient in tablets

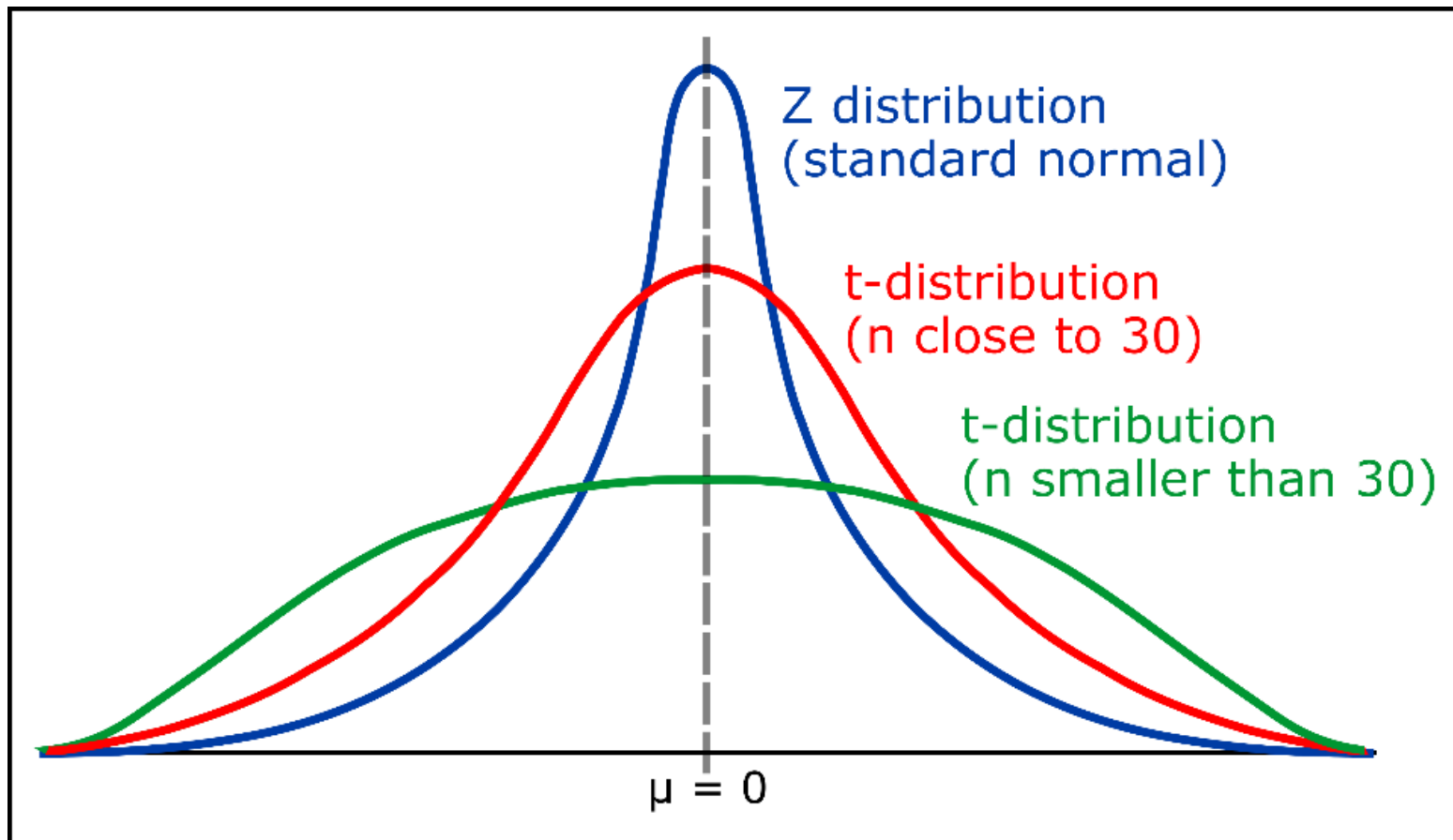
- Based on a sample of 100 tablets you learn your baseline milli-grams of active ingredient is 250 mg with a Standard Deviation of 10 mg.
- Because this is just a sample the 250 mg average is an estimate of the true average.
- Calculate the **95% CONFIDENCE INTERVAL CI**

$$\text{95\% CI : } \bar{X} \pm 2 \frac{S_x}{\sqrt{n}} = 250 \text{ mg} \pm 2 \frac{10}{\sqrt{40}} = 250 \text{ mg} \pm 6.32$$

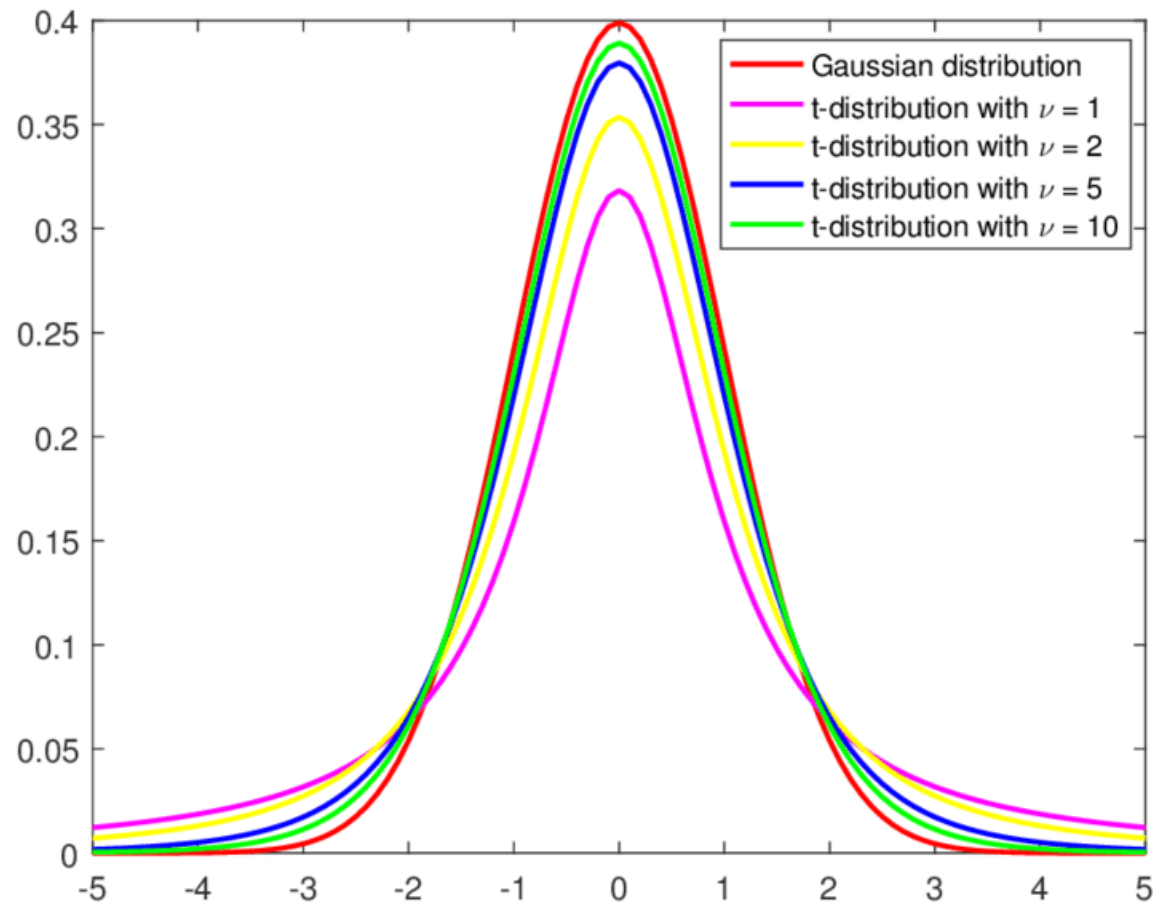
Conclusion:

We can say with 95% confidence that the true average is somewhere between 243.68 and 256.32 mg

Diagram

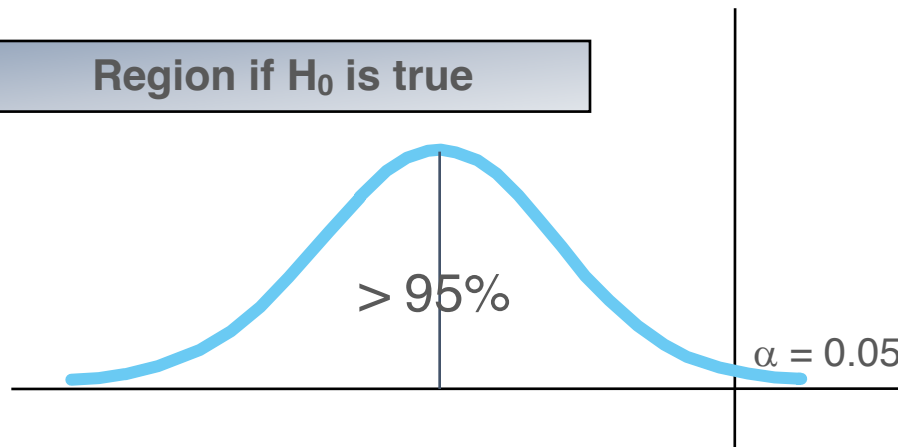


Diagram



One Sided Risk Vs Two Sided Risk

Region if H_0 is true

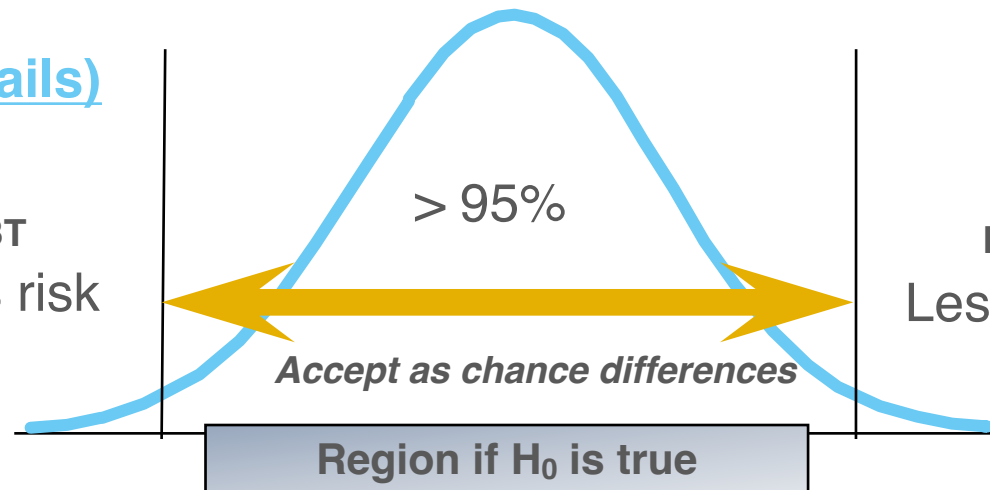


One sided risk (1 tail)

Region of DOUBT
Less than 5% risk

2 sided risk (2 tails)

Region of DOUBT
Less than 2.5% risk



Region of DOUBT
Less than 2.5% risk

Table 1

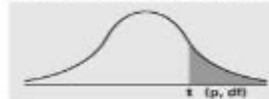
T Table 1

CONFIDENCE LEVEL

	0.600	0.700	0.800	0.900	0.950	0.975	0.990	0.995
DF = n-1								
1	0.325	0.727	1.376	3.078	6.314	12.706	31.821	63.657
2	0.289	0.617	1.061	1.886	2.920	4.303	6.965	9.925
3	0.277	0.584	0.978	1.638	2.353	3.182	4.541	5.841
4	0.271	0.569	0.941	1.533	2.132	2.776	3.747	4.604
5	0.267	0.559	0.920	1.476	2.015	2.571	3.365	4.032
6	0.265	0.553	0.906	1.440	1.943	2.447	3.143	3.707
7	0.263	0.549	0.896	1.415	1.895	2.365	2.998	3.499
8	0.262	0.546	0.889	1.397	1.860	2.306	2.896	3.355
9	0.261	0.543	0.883	1.383	1.833	2.262	2.821	3.250
10	0.260	0.542	0.879	1.372	1.812	2.228	2.764	3.169
11	0.260	0.540	0.876	1.363	1.796	2.201	2.718	3.106
12	0.259	0.539	0.873	1.356	1.782	2.179	2.681	3.055
13	0.259	0.538	0.870	1.350	1.771	2.160	2.650	3.012
14	0.258	0.537	0.868	1.345	1.761	2.145	2.624	2.977
15	0.258	0.536	0.866	1.341	1.753	2.131	2.602	2.947
16	0.258	0.535	0.865	1.337	1.746	2.120	2.583	2.921
17	0.257	0.534	0.863	1.333	1.740	2.110	2.567	2.898
18	0.257	0.534	0.862	1.330	1.734	2.101	2.552	2.878
19	0.257	0.533	0.861	1.328	1.729	2.093	2.539	2.861
20	0.257	0.533	0.860	1.325	1.725	2.086	2.528	2.845
21	0.257	0.532	0.859	1.323	1.721	2.080	2.518	2.831
22	0.256	0.532	0.858	1.321	1.717	2.074	2.508	2.819
23	0.256	0.532	0.858	1.319	1.714	2.069	2.500	2.807
24	0.256	0.531	0.857	1.318	1.711	2.064	2.492	2.797
25	0.256	0.531	0.856	1.316	1.708	2.060	2.485	2.787
26	0.256	0.531	0.856	1.315	1.706	2.056	2.479	2.779
27	0.256	0.531	0.855	1.314	1.703	2.052	2.473	2.771
28	0.256	0.530	0.855	1.313	1.701	2.048	2.467	2.763
29	0.256	0.530	0.854	1.311	1.699	2.045	2.462	2.756

Table 2

Numbers in each row of the table are values on a t -distribution with (df) degrees of freedom for selected right-tail (greater-than) probabilities (p).



df/p	0.40	0.25	0.10	0.05	0.025	0.01	0.005	0.0005
1	0.324920	1.000000	3.077684	6.313752	12.70620	31.82052	63.65674	636.6192
2	0.288675	0.816497	1.885618	2.919986	4.30285	6.96456	9.92484	31.5991
3	0.276671	0.764892	1.637744	2.353363	3.18245	4.54070	5.84091	12.9240
4	0.270722	0.740697	1.533206	2.131847	2.77645	3.74695	4.60409	8.6103
5	0.267181	0.726687	1.475884	2.015048	2.57058	3.36493	4.03214	6.8688
6	0.264835	0.717558	1.439756	1.943180	2.44691	3.14267	3.70743	5.9588
7	0.263167	0.711142	1.414924	1.894579	2.36462	2.99795	3.49948	5.4079
8	0.261921	0.706387	1.396815	1.859548	2.30600	2.89646	3.35539	5.0413
9	0.260955	0.702722	1.383029	1.833113	2.26216	2.82144	3.24984	4.7809
10	0.260185	0.699812	1.372184	1.812461	2.22814	2.76377	3.16927	4.5869
11	0.259556	0.697445	1.363430	1.795885	2.20099	2.71808	3.10581	4.4370
12	0.259033	0.695483	1.356217	1.782288	2.17881	2.68100	3.05454	4.3178
13	0.258591	0.693829	1.350171	1.770933	2.16037	2.65031	3.01228	4.2208
14	0.258213	0.692417	1.345030	1.761310	2.14479	2.62449	2.97684	4.1405
15	0.257885	0.691197	1.340606	1.753050	2.13145	2.60248	2.94671	4.0728
16	0.257599	0.690132	1.336757	1.745884	2.11991	2.58349	2.92078	4.0150
17	0.257347	0.689195	1.333379	1.739607	2.10982	2.56693	2.89823	3.9651
18	0.257123	0.688364	1.330391	1.734064	2.10092	2.55238	2.87844	3.9216
19	0.256923	0.687621	1.327728	1.729133	2.09302	2.53948	2.86093	3.8834
20	0.256743	0.686954	1.325341	1.724718	2.08596	2.52798	2.84534	3.8495
21	0.256580	0.686352	1.323188	1.720743	2.07961	2.51765	2.83136	3.8193
22	0.256432	0.685805	1.321237	1.717144	2.07387	2.50832	2.81876	3.7921
23	0.256297	0.685306	1.319460	1.713872	2.06866	2.49987	2.80734	3.7676
24	0.256173	0.684850	1.317836	1.710882	2.06390	2.49216	2.79694	3.7454
25	0.256060	0.684430	1.316345	1.708141	2.05954	2.48511	2.78744	3.7251
26	0.255955	0.684043	1.314972	1.705618	2.05553	2.47863	2.77871	3.7066
27	0.255858	0.683685	1.313703	1.703288	2.05183	2.47266	2.77068	3.6896
28	0.255768	0.683353	1.312527	1.701131	2.04841	2.46714	2.76326	3.6739
29	0.255684	0.683044	1.311434	1.699127	2.04523	2.46202	2.75639	3.6594
30	0.255605	0.682756	1.310415	1.697261	2.04227	2.45726	2.75000	3.6460
z	0.253347	0.674490	1.281552	1.644854	1.95996	2.32635	2.57583	3.2905
CI	———	———	80%	90%	95%	98%	99%	99.9%

Small Sample Sizes : T- Distribution of Sample Averages

Small Sample Sizes =
T- Distribution of Sample Averages

The probability is 95% that the sample Mean is within $\pm t$ standard errors of the true mean of the population.

The value of **t** changes with Sample Size

Practical Example – Small Sample Size

- Based on a sample of **5** tablets you learn your baseline grams of active ingredient is 250 mg with a Standard Deviation of 10 mg.
- Since it is small sample size we use t-distribution
- Degrees of Freedom $n - 1 = 4$
 - - $t = 2.776$

$$\mathbf{95\% \text{ CI : } \quad \bar{X} \pm t \frac{S_x}{\sqrt{n}}}$$

$$= 250 \text{ mg} \pm t \frac{10}{\sqrt{5}} = 250 \text{ mg} \pm \underline{\hspace{2cm}}$$

Conclusion:

We can say with 95% confidence that the true average milligrams of active ingredient is between _____ and _____

Practical Example – Small Sample Size

- Based on a sample of **15** reviews, you learn that your baseline average time taken to review a Credit Card application is **45 minutes** with a Standard Deviation of **10 minutes**.
- Since it is small sample size we use t-distribution
- Degrees of Freedom $n - 1 = 14$
- $t = 2.145$

$$\mathbf{95\% \text{ CI} : } \bar{X} \pm t \frac{S_x}{\sqrt{n}}$$

$$= 45 \pm t \frac{10}{\sqrt{15}} = 45 \text{ minutes} \pm \underline{\hspace{2cm}}$$

Conclusion:

We can say with **95% confidence** that the true average time is between

Summary

At this point you should be able to:

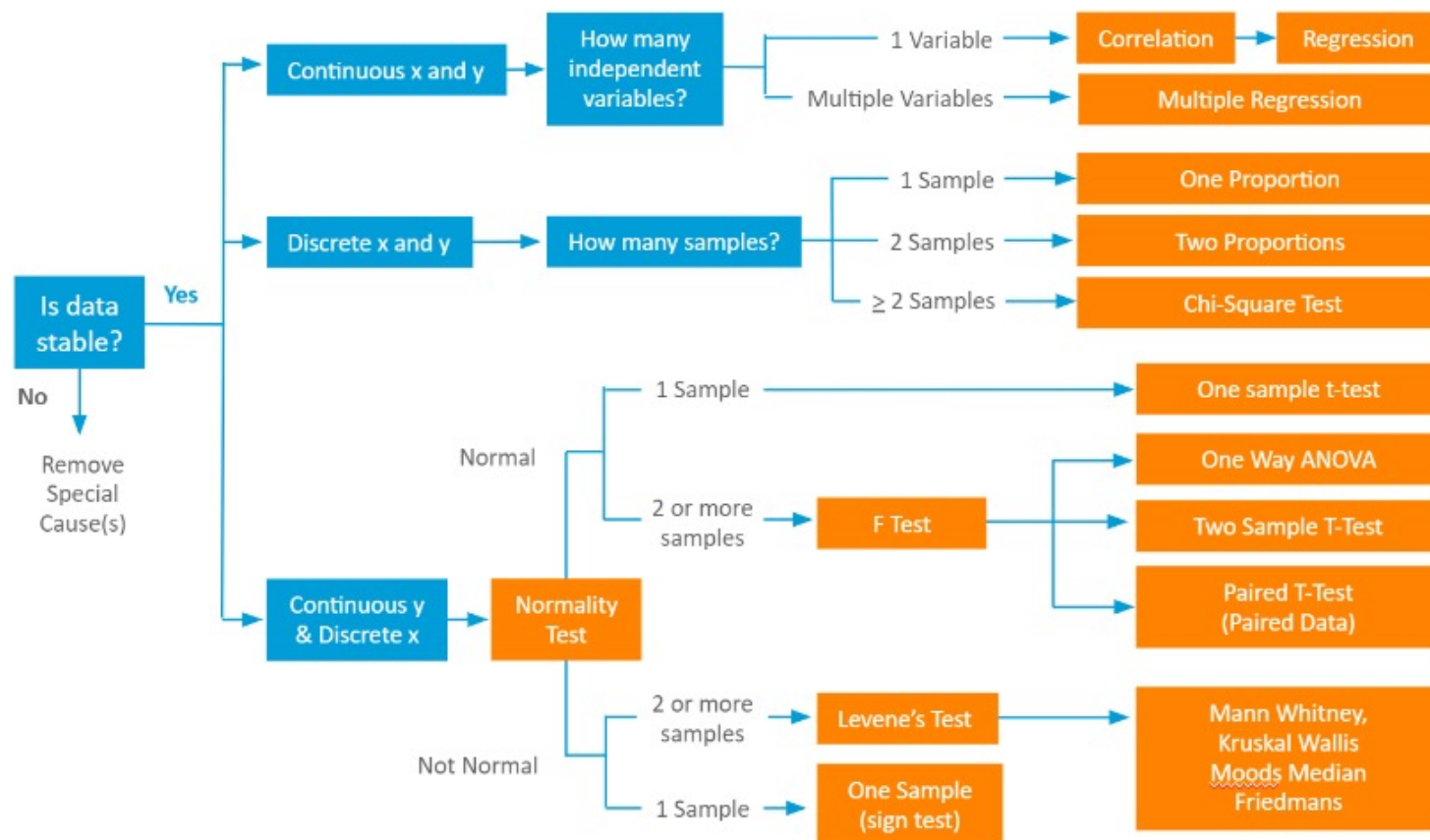
- Explain the term “Inferential Statistics”
- Explain the Central Limit Theorem
- Describe what impact sample size has on your estimates of population parameters
- Understand the difference between a Normal Distribution and t-Distribution
- Explain Standard Error of the Mean (SEM)

Advanced Hypothesis Testing

Continuous Normal Data

T-Tests

Hypothesis Testing Roadmap



1 – Sample T-test Sample Size

Population

n = 7

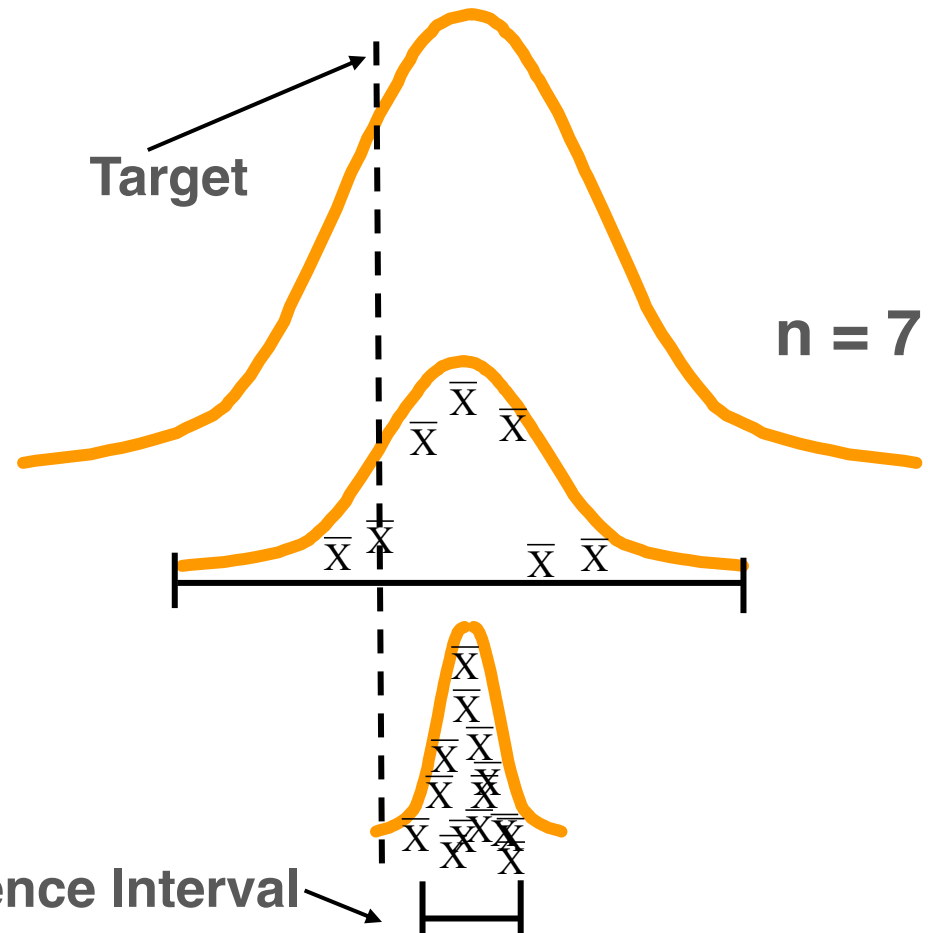
Cannot tell the difference between the sample and the target.

n = 20

Can tell the difference between the sample and the target.

$$SE\ Mean = \frac{S}{\sqrt{n}}$$

Confidence Interval



P-Values

Any differences between observed data and claims made under H_0 may be real or due to chance.

Hypothesis Tests determine the probabilities of these differences occurring *solely due to chance* and call them **P-values**.

The **α level** of a test (*level of significance*) represents the yardstick against which *P-values* are measured and H_0 is rejected if the ***P-value*** is less than the alpha level.

Alpha level= 0.05

Hypothesis Testing – Type 1 Error

Type 1 Error (also called the “**False Positive**”) is the probability we could be wrong in saying two or more things are different when, in fact, they are the same.

In other words alpha risk is the risk that we accept the alternative when in fact the null is correct

		<i>Actual Conditions</i>	
		Not Different (H_0 is True)	Different (H_0 is False)
<i>Statistical Conclusions</i>	Not Different (Fail to Reject H_0)	Correct Decision	
	Different (Reject H_0)	Type 1 Error	Correct Decision

Hypothesis Testing – Type 2 Error

Type 2 Error (also called the “**False Negative**”) is the probability we could be wrong in saying two or more things are the same when, in fact, they are different.

In other words beta risk is the risk that we accept the null when in fact the alternative is correct

	<i>Actual Conditions</i>	
	Not Different (H_0 is True)	Different (H_0 is False)
<i>Statistical Conclusions</i>	Not Different (Fail to Reject H_0)	Correct Decision
	Different (Reject H_0)	Type 2 Error
	Type 1 Error	Correct Decision

Confidence Level = $1 - \alpha$ | Power = $1 - \beta$

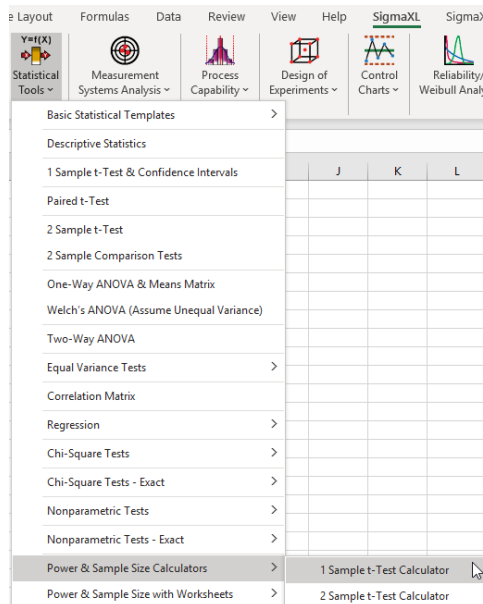
- **“Power”** is the word SigmaXL uses for describing the test or study. “Power that the test/study will correctly detect the alternative if the alternative is in fact true ”
- Power = $1 - \beta$
- (Beta level of risk is the probability that you accept the null even though the alternative was correct)

- **“Confidence Level”** is the word SigmaXL uses for describing the level of confidence you have in the NULL hypothesis.
- “Confidence Level that you will correctly reject the null when the null is incorrect.
- Confidence Level = $1 - \alpha$ level
- (Alpha level of risk is the probability that you reject the null even though the null is correct)

Sample Size

Example : Standard deviation in a sample of parts is is 0.1 mm

You want to be able to detect differences a low as 0.075 mm. What is the Minimum sample size your must test to have a test with 90% Power to correctly Detect the difference of 0.075 mm.



Power and Sample Size: 1 Sample t-Test Calculator

Solve For

Power (1-Beta) ☐

Sample Size (N)

Difference (Mean - Ref) ☐

Standard Deviation

Significance Level (Alpha)

Ha:

Buttons: OK >>, Cancel, Help

Three fields must be filled in and one left blank.

Sample Size

SigmaXL tells us we need a minimum sample size of 21 samples to have a test with 90% Power to correctly detect the difference of 0.075 mm.

Power and Sample Size: 1 Sample t Test					
H0: Mean (μ) = Reference					
Ha: Mean (μ) $\neq$ Reference					
Solve For: Sample Size (N)					
Power (1 - Beta)	Difference	Standard Deviation	Significance Level (Alpha)	Sample Size (N)	Actual Power
0.9	0.075	0.1	0.05	21	0.904533362

Sample Size

- It is common to state the difference in terms of “generic” Standard Deviations when you do not have an estimate for the Standard Deviation of the process.
 - For example, if you want to detect a shift of 1.5 Standard Deviations enter that in “Differences” and enter 1 for “Standard deviation”.
 - If you knew the Standard Deviation was 0.8 then enter it and 1.2 for the “Differences” (which is a 1.5 Standard Deviation shift in terms of real values).
 - If you are unsure of the desired difference, or in many cases simply get stuck with a sample size you did not have a lot of control over, SigmaXL will tell you how much of a difference can be detected.
- You as a practitioner must be careful when drawing Practical Conclusions because it is possible to have statistical significance without practical significance.
 - In other words - do a reality check. SigmaXL has made it easy to see an assortment of sample sizes and differences.

1 – Sample t Example

SigmaXL > Statistical Tools > Power and Sample Size Calculators > 1 Sample t-test Calculator

Power and Sample Size: 1 Sample t-Test Calculator

Solve For

Power (1-Beta)

Sample Size (N)

Difference (Mean - Ref)

Standard Deviation

Significance Level (Alpha)

Ha:

OK >> Cancel Help

This means we will be able to detect a difference of only 1.24 if the population has a Standard Deviation of 1 unit.

SigmaXL Results

Power and Sample Size: 1 Sample t Test				
H0: Mean (μ) = Reference				
Ha: Mean (μ) $\neq$ Reference				
Solve For: Difference (Mean - Reference)				
Power (1 - Beta)	Sample Size (N)	Standard Deviation	Significance Level (Alpha)	Difference
0.9	9	1	0.05	1.237

1 – Sample t Exercise

Exercise objective:

Conduct and analyze a one sample t-test using SigmaXL.

1.The engineering estimation claimed we would achieve a chemical with average active ingredient of 32 parts per million (ppm).

2.We want to test if we are achieving this performance level, we want to know if we are on target with 95% confidence in our answer. Use worksheet **HYPOTTESTSTUD** with data in column **ppm VOC**

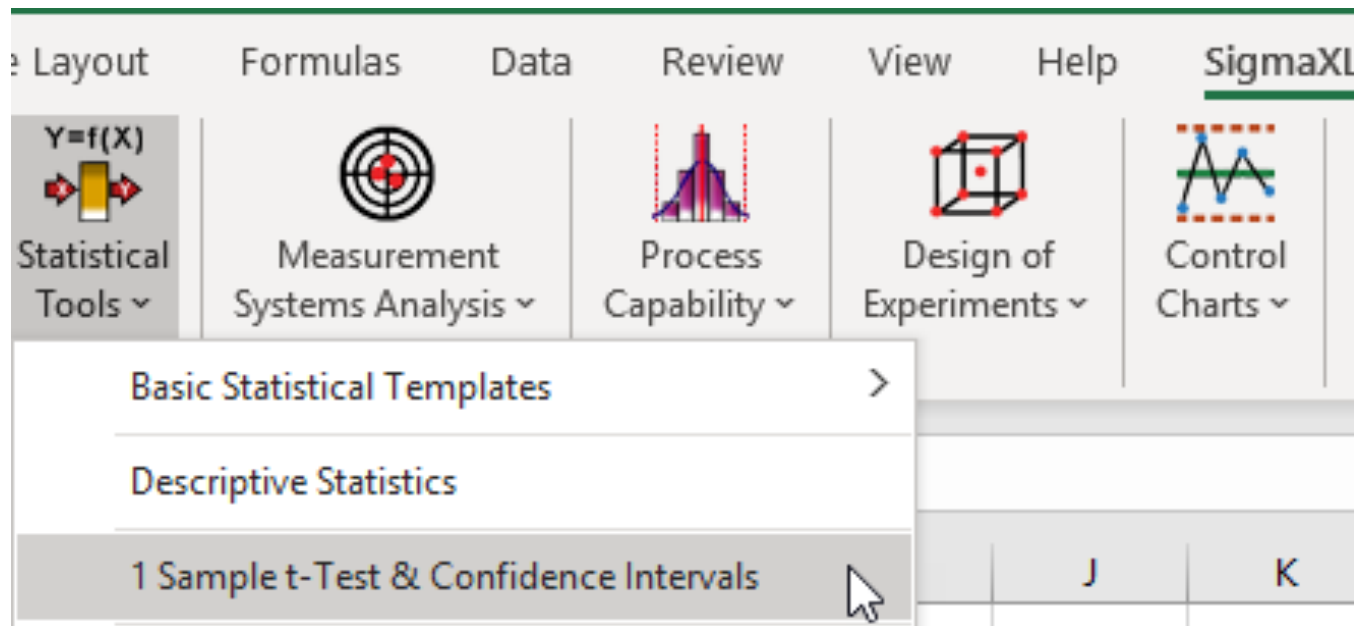
3. Are we on Target?

7 Steps to Statistical Hypothesis

1. State the Practical Problem.
2. State the Statistical Problem.
 - a) $H_0: \text{---} = \text{---}$
 - b) $H_A: \text{---} \neq, >, < \text{---}$
3. Run a Normality test on the data (**Use Anderson-Darling Test or Shiro-Wilk**)
4. Select the appropriate Hypothesis test and risk levels.
5. Run statistical test on the data
6. State the Statistical Solution.
7. State the Practical Solution.

1 – Sample t Exercise: Solution

We will use the 1 sample T test to determine if we are at Target.



1 – Sample t Exercise: Solution

Select column ppm VOC and set Null Hypothesis mean to 32.0.

1 Sample t-Test

Shift
RM Supplie
Days to Deliver

☒ **S**tacked Column Format (1 Numeric Data Column & 1 Optional Group Category)
☐ **U**nstacked Column Format (1 or more Numeric Data Columns)

Numeric Data Variable (Y) >> ppm VOC

Optional Group Category (X) >>

<< **R**emove

H0: Mean = 32

Ha: Not Equal To

Confidence Level: 95.0

☐ Display Test Assumptions Report

OK >>
Cancel
Help

1 – Sample t Exercise: Solution

In the SigmaXL results tab you can see the P-value of 0.201. Because it is above 0.05 we “fail to reject” the null hypothesis so we accept the equipment is giving product at a target of 32.0 ppm VOC.

1 Sample t-Test	
Test Information	
H_0 : Mean (Mu) = 32	
H_a : Mean (Mu) Not Equal To 32	
Results:	ppm VOC
Count	24
Mean	34.042
StDev	7.601
SE Mean	1.552
t	1.316
P-Value (2-sided)	0.2012
UC (2-sided, 95%)	37.251
LC (2-sided, 95%)	30.832

F - Tests

- The test is based on F-Distribution and named after the Statistician Ronald Fisher (1890 – 1962)
- Tests whether 2 populations have equal variance
- 2 independently collected samples of 2 populations that follow normal distribution (a PARAMETRIC TEST).
- Also plays an important role in the Analysis of Variance tool (ANOVA) which tests if several population MEANS are the same or different.
- Also used to check if the data conforms to a regression model, which is acquired through a method called 'Least square analysis.'
- Also used in Measurement System Analysis tests.
- For full mathematical explanation see :
<https://www.youtube.com/watch?v=VwQX1I5SrP4> (Not needed on exam)

Comparison Between T - Test and F - Test

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Comparison Between T - Test and F - Test

BASIS FOR COMPARISON	T-TEST	F-TEST
Meaning	T-test is a univariate hypothesis test, that is applied when standard deviation is not known and the sample size is small.	F-test is statistical test, that determines the equality of the variances of the two normal populations.
Test statistic	T-statistic follows Student t-distribution, under null hypothesis.	F-statistic follows Snedecor f-distribution, under null hypothesis.
Application	Comparing the means of two populations.	Comparing two population variances.

F – Test to Compare Two Variants

1. Formulate the null and alternate hypotheses.

$$H_0: s_1^2 = s_2^2$$

$$H_a: s_1^2 > s_2^2$$

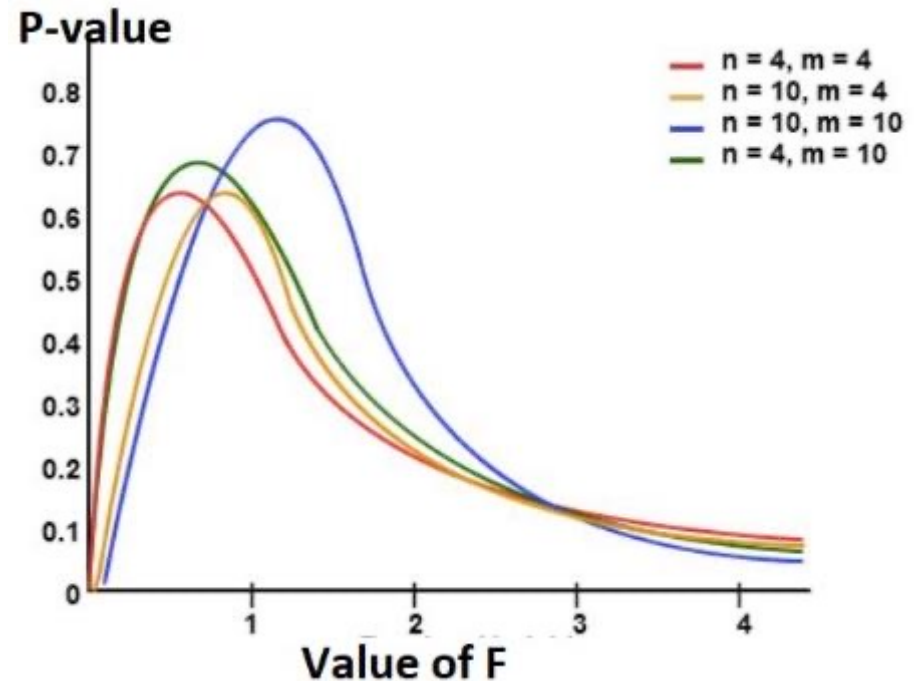
[Note that we might also use $s_1^2 < s_2^2$ or $s_1^2 \neq s_2^2$]

2. Run F-Test in SigmaXL (Variance Test or Standard Deviation Test)
 - SigmaXL > Templates and Calculators > Basic Statistical Templates > 2 Sample F-Test and CI
3. Reject the null hypothesis of equal population variances if **p-value < Alpha Level of Risk**

F – Distribution PDF

n = size of sample 1
m = size of sample 2

F - Distribution becomes more Normal as **n** and **m** increase (less skewed)



F – Test Example

The variability in the **amount of impurities** present in a batch of chemicals used for a process depends on the length of time that the process is in operation.

Suppose a sample of size 10 is drawn from the normal process which is to be compared to a sample of a new process that has been developed to reduce the **variability of impurities**.

	Sample 1	Sample 2
n	10	10
StDev	1.02	0.714

Sample F - Test

SigmaXL > Templates and Calculators > Basic Statistical Calculators > 2 Sample F-Test and CI



2 Sample F-Test and Ratio Confidence Interval (Compare 2 Standard Deviations)

Sample Data (user inputs):		Sample 1	Sample 2
Sample Size	n	10	10
Sample Standard Deviation	s	1.0200	0.7140
Null Hypothesis (hypothesized ratio of StDevs)	$H_0: \sigma_1/\sigma_2 =$	1.0	
Alternative Hypothesis	$H_a: \sigma_1/\sigma_2$	Not Equal To	
Confidence Level (enter .95 for 95%)		95.0%	
Results:			
Ratio of Standard Deviations		1.4286	
Sample Variance		1.0404	0.5098
Ratio of Variances		2.0408	
df		9	9
F-statistic		2.0408	
alpha		0.0500	
P-Value (2-sided)		0.3028	
Upper Confidence Limit (2 Sided)		2.8664	
Lower Confidence Limit (2 Sided)		0.7120	

Purpose of ANOVA

Analysis of Variance (ANOVA) extends the two sample t-test for testing the equality of two population Means to a more general hypothesis of comparing the equality of more than two Means.

There are several types of ANOVA hypothesis test depending on how much INTERACTION you want to understand between samples:

- **1-way ANOVA (does not look for interaction effects)**
- **2-way ANOVA (looks for 2 way interaction effects)**
- **Balanced ANOVA (complex 3-way and more interactions)**
- **Nested ANOVA (complex 3-way and more interactions)**
- **Multivariate ANOVA (MANOVA - even more complex !!)**

1 – Way ANOVA With Three Samples

We have three potential suppliers claiming to have equal levels of quality. Supplier B provides a considerably lower purchase price than either of the other two vendors. We would like to choose the lowest cost supplier but we must ensure we do not effect the quality of our raw material. Assume no interaction effects.

A	B	C
Supplier A	Supplier B	Supplier C
3.16	4.24	4.58
4.35	3.87	4
3.46	3.87	4.24
3.74	4.12	3.87
3.61	3.74	3.46

We would like test the data to determine if there is a difference between the three suppliers.

SigmaXL Boxplot

The screenshot displays the SigmaXL software interface. The top menu bar includes 'aw', 'Page Layout', 'Formulas', 'Data', 'Review', 'View', 'Help', and 'SigmaXL'. The 'Graphical Tools' dropdown menu is open, showing options like 'Basic Graphical Templates', 'Basic Pareto Chart', 'Advanced Pareto Charts', 'Advanced Pareto Options', 'EZ Pivot/Pivot Charts', 'Basic Histogram', 'Histograms & Descriptive Statistics', 'Histograms & Process Capability', 'Dotplots', and 'Boxplots'. The 'Boxplots' option is selected. The 'Boxplots' dialog box is open, showing a list of 'Numeric Data Variables (Y)' with 'Supplier A', 'Supplier B', and 'Supplier C' listed. The 'Group Category (X1)' and 'Group Category (X2)' fields are empty. The 'Show Mean' checkbox is checked, and the 'Show Legend' and 'Add Title' checkboxes are unchecked. The 'OK >>' button is highlighted.

aw Page Layout Formulas Data Review View Help SigmaXL

Graphical Tools Statistical Tools Measurement Systems Analysis Process Capability Design of Experiments Control Charts

Basic Graphical Templates >

Basic Pareto Chart

Advanced Pareto Charts

Advanced Pareto Options

EZ Pivot/Pivot Charts

Basic Histogram

Histograms & Descriptive Statistics

Histograms & Process Capability

Dotplots

Boxplots

Boxplots

Numeric Data Variables (Y) >>

Supplier A
Supplier B
Supplier C

OK >>

Cancel

Help

Group Category (X1) >>

Group Category (X2) >>

<< Remove

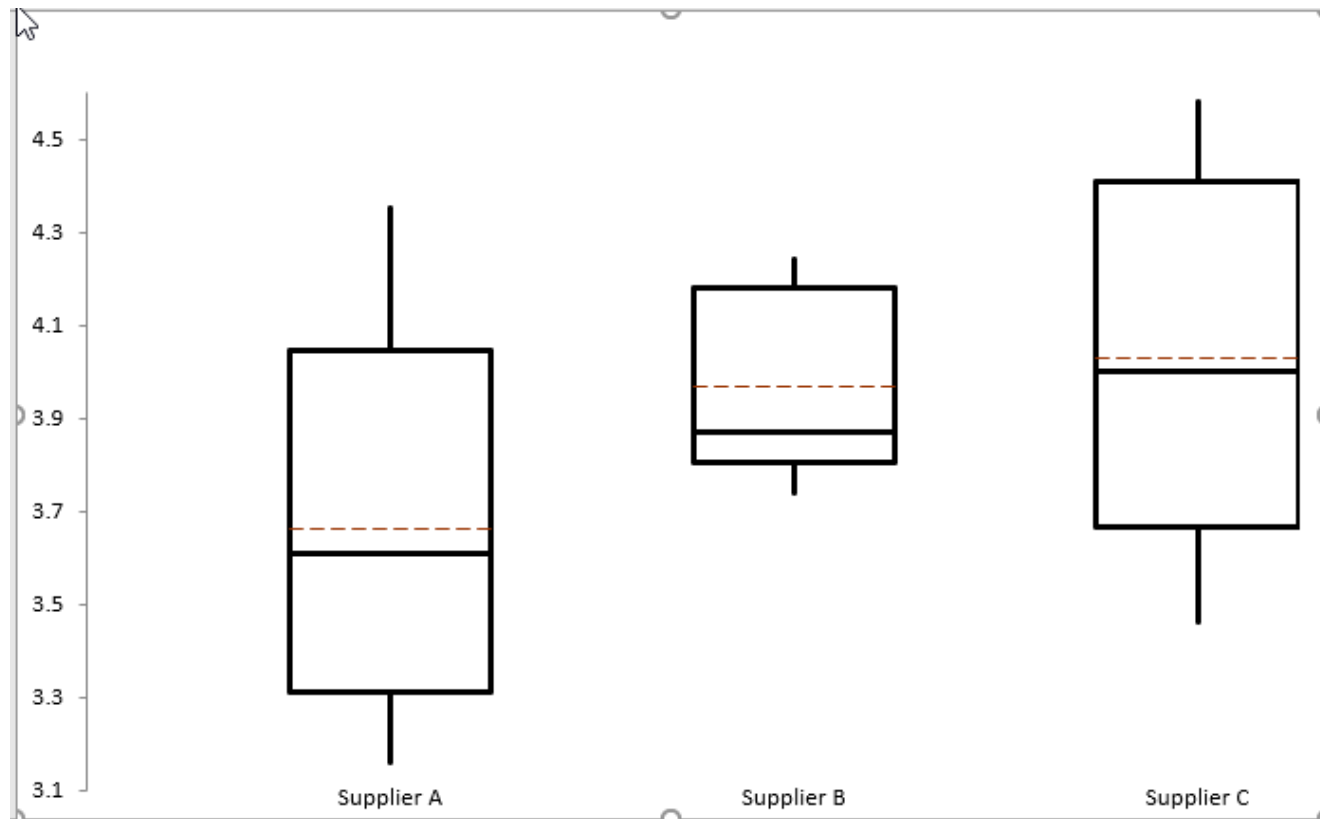
☒ Show Mean

☐ Show Legend

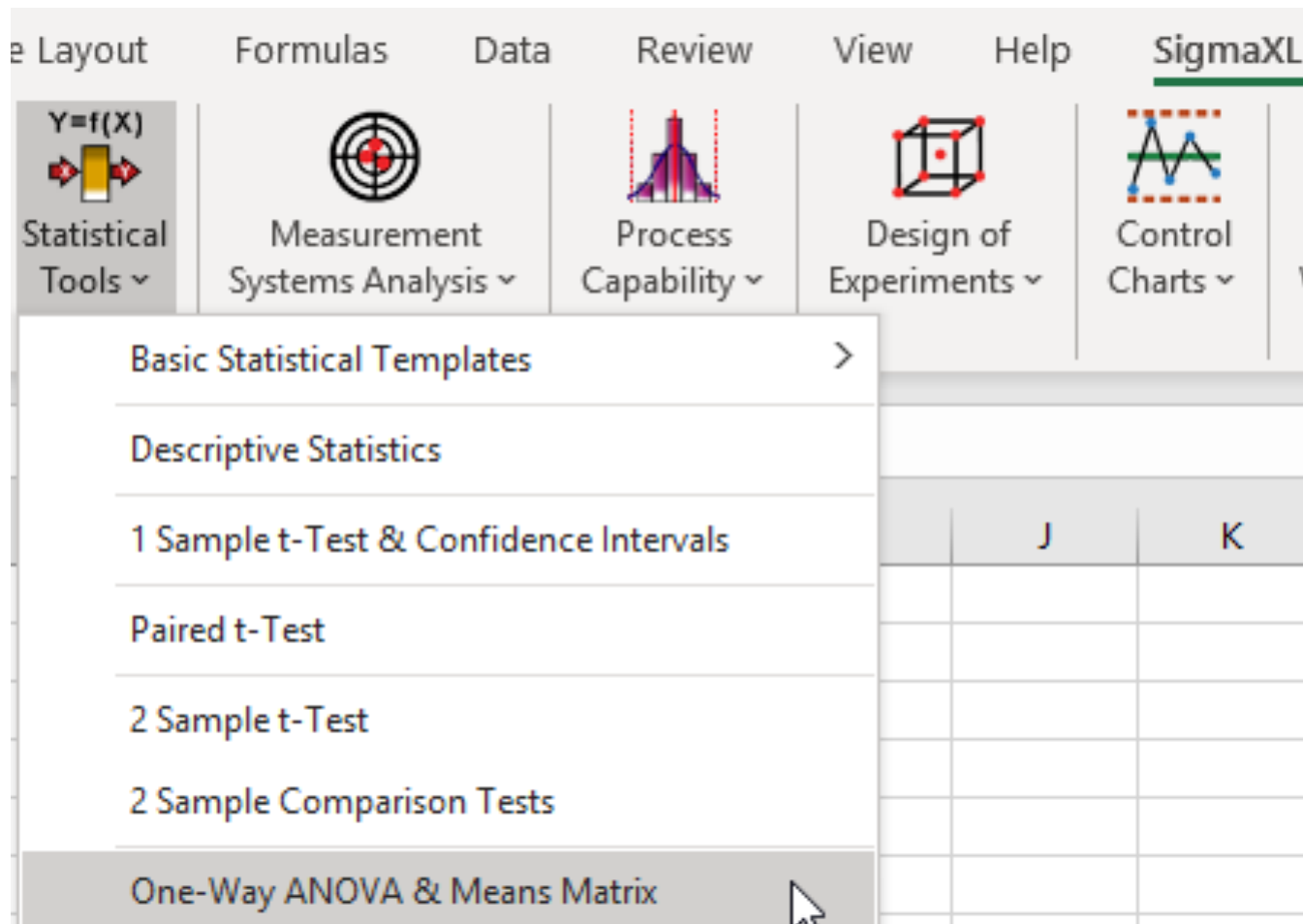
☐ Add Title

SigmaXL Boxplot

What does this graph suggest to us?



One Way ANOVA

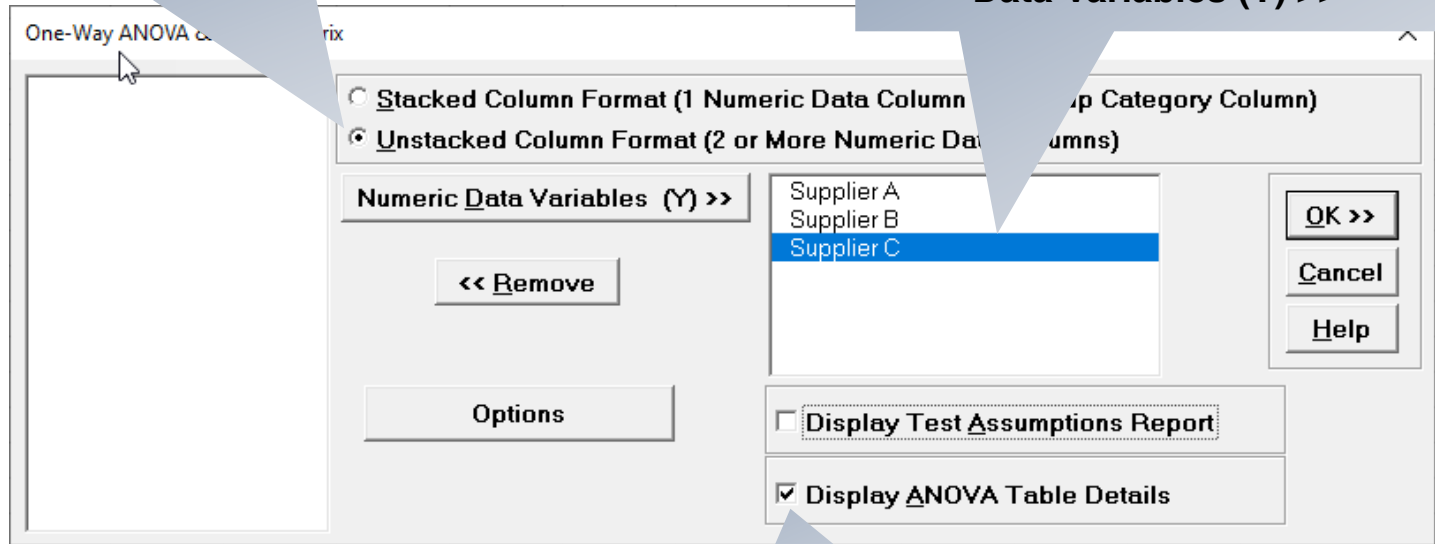


ANOVA SigmaXL

Statistical Tools > One-Way ANOVA & Means Matrix

Check “Unstacked Column Format (2 or More Numeric Data Columns)”

Enter Supplier data in “Numeric Data Variables (Y) >>”



Check “Display ANOVA Table Details”

The screenshot shows the 'One-Way ANOVA & Means Matrix' dialog box. The 'Unstacked Column Format (2 or More Numeric Data Columns)' option is selected. The 'Numeric Data Variables (Y) >>' list contains 'Supplier A', 'Supplier B', and 'Supplier C'. The 'Display ANOVA Table Details' checkbox is checked. The 'Display Test Assumptions Report' checkbox is unchecked. The 'Options' button is visible at the bottom left of the dialog.

ANOVA Session Window

P-value > .05
No Difference between suppliers

ANOVA Table						
Source	SS	DF	MS	F	P-Value	
Between	0.383693	2	0.191847	1.403	0.2835	
Within	1.641	12	0.136767			
Total	2.025	14				

6. State Statistical Conclusions: $p\text{-value} > 0.05$
Accept the null hypothesis

7. State Practical Conclusions: We cannot say that there is any difference between the 3 suppliers.

ANOVA Exercise



Exercise objective: Utilize what you have learned to conduct an analysis of a one way ANOVA using SigmaXL.

The quality manager was challenged by the plant director as to why the VOC levels in the product varied so much. The quality manager now wants to find if the product quality is different owing to how the shifts work with the product.

Exercise - Solution

1. **Practical Problem:** _____
2. **Statistical Problem:** _____
3. **Test for Normality. Is the data Normal ?** _____
4. **Choose the correct statistical test:** _____
5. **Run the test on the data**
6. **State Statistical Conclusions:** _____
7. **State Practical Conclusions:** _____

Hypothesis Testing

Non Normal Data

Non-Parametric Testing

- Non-Normal tests are called ‘Non-Parametric’ tests in SigmaXL
- Non-parametric Hypothesis Testing works the same way as parametric testing.
- Evaluate the p-value in the same manner as Parametric tests.



Non-Parametric Methods

In general, for a statistical method to be classified as nonparametric, it must satisfy at least one of the following conditions.

- The method can be used with **nominal data**.
- The method can be used with **ordinal data**.
- The method can be used with interval or ratio data when no assumption can be made about the population probability distribution.
- (No assumption that it is a Normal Distribution)

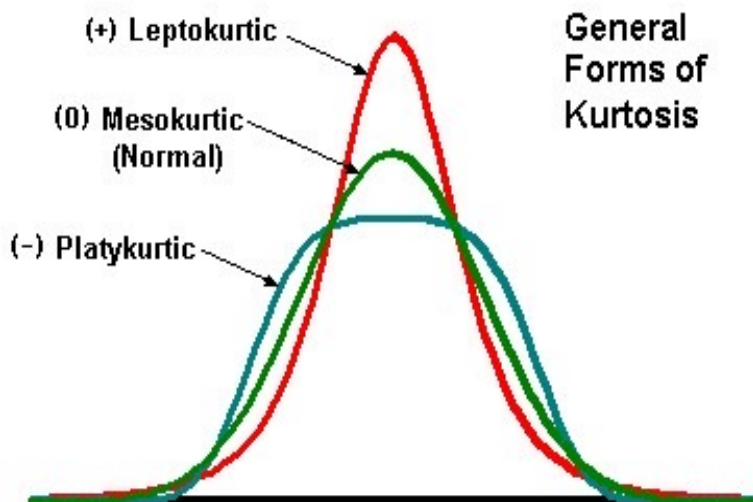
Scales of Measurement (Green Belt)

- **N**ominal scale: A nominal scale is really a list of categories to which objects can be classified.
- **O**rdinal scale: is a measurement scale that assigns values to objects based on their ranking with respect to one another.
- **I**nterval scale: one unit on the scale represents the same magnitude on the trait or characteristic being measured across the whole range of the scale. For example, if anxiety were measured on an interval scale, then a difference between a score of 10 and a score of 11 would represent the same difference in anxiety as would a difference between a score of 50 and a score of 51.
- **R**atio scale: Ratio scales are like interval scales except they have true zero points.

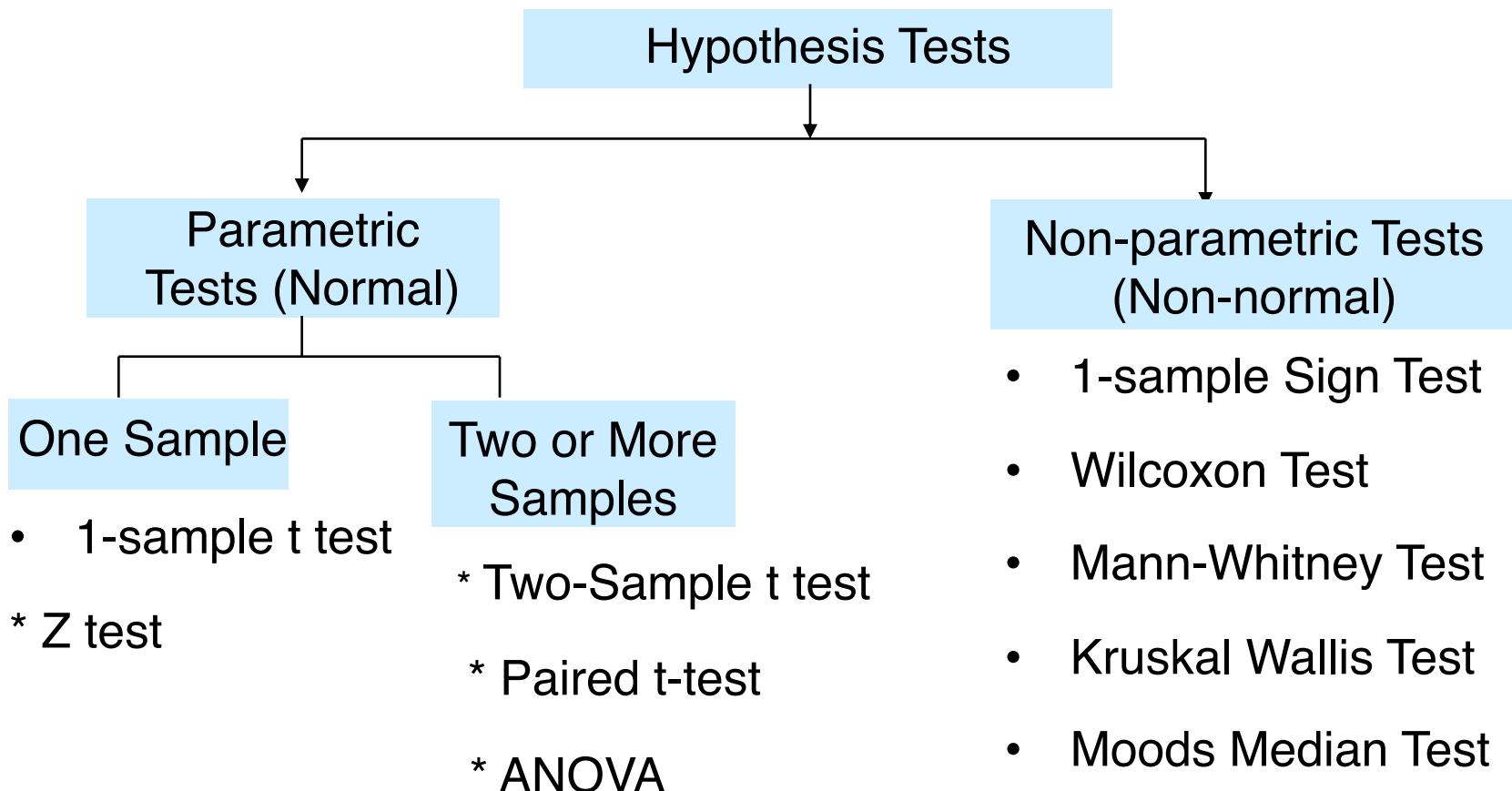
Scales of Measurement (Green Belt)

Skewness is a measure of symmetry, or more precisely, the lack of symmetry. A distribution, or data set, is symmetric if it looks the same to the left and right of the center point.

Kurtosis is a measure of whether the data are peaked or flat relative to a normal distribution. Data sets with low kurtosis tend to have a flat top near the mean rather than a sharp peak. A statistical measure used to describe the distribution of observed data around the mean



Parametric and Non-Parametric



SigmaXL's Non-Parametric Test

1-Sample Sign: performs a one-sample sign test of the Median and calculates the corresponding point estimate and confidence interval. Use this test as an alternative to one-sample Z and one-sample t-tests.

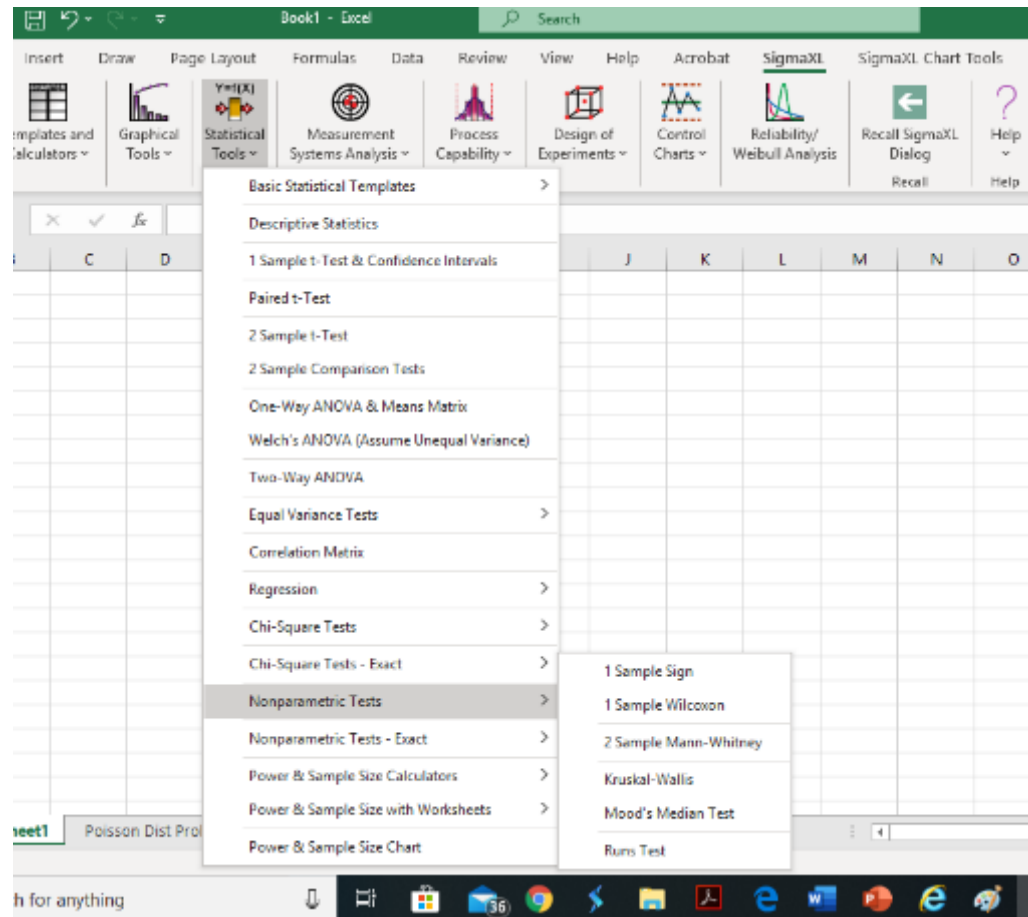
1-Sample Wilcoxon: performs a one-sample Wilcoxon signed rank test of the Median and calculates the corresponding point estimate and confidence interval (more discriminating or efficient than the sign test). Use this test as a nonparametric alternative to one-sample Z and one-sample t-tests.

Mann-Whitney: performs a Hypothesis Test of the equality of two population Medians and calculates the corresponding point estimate and confidence interval. Use this test as a nonparametric alternative to the two-sample t-test.

Kruskal-Wallis: performs a Hypothesis Test of the equality of population Medians for more than 2 samples. This test is more powerful than Mood's Median (the confidence interval is narrower, on average) for analyzing data from many populations, but is less robust to outliers. Use this test as an alternative to the one-way ANOVA.

Mood's Median Test: performs a Hypothesis Test of the equality of population Medians for more than 2 samples. Test is similar to the Kruskal-Wallis Test.

Non-Parametric Tests in SigmaXL



Sign Test – Customer Satisfaction

Example: Customer Satisfaction Score

- Historically our Median customer satisfaction score has been 3.5.
- We would like to see if this has changed, with the results grouped by customer type
 - H_0 : Median = 3.5
 - H_a : Median $\neq$ 3.5
 - $\alpha = 0.05$

Sign Test – Customer Satisfaction

Hypotheses

H0: Median=3.5, Ha: Median $\neq$ 3.5, $\alpha = 0.05$

SigmaXL > Statistical Tools > Nonparametric Tests > 1 Sample Sign

Nonparametrics: 1 Sample Sign Test

Customer Record No
Order Date
Avg No. of orders per mo
Avg days Order to delivery
Loyalty - Likely to Recomm
Responsive to Calls
Ease of Communications
Staff Knowledge
Size of Customer
Major-Complaint
Product Type
Sat-Discrete

☒ **Stacked Column Format (1 Numeric Data Column & 1 Optional Group Category)**
☐ **Unstacked Column Format (1 or more Numeric Data Columns)**

Numeric Data Variable (Y) >> Overall Satisfaction

Optional Group Category (X) >> Customer Type

<< Remove

H0: Median = 3.5

Ha: Not Equal To

OK >>
Cancel
Help



Sign Test – Customer Satisfaction

Results

- Note the P-Values.
- Customer Type 2 shows a significant change (increase) in Satisfaction Median (P-Value < .05), whereas Customer Types 1 and 3 show no change (P-Value ≥ .05).

1 Sample Sign Test for Medians: Overall Satisfaction				
Test Information				
H ₀ : Median = 3.5				
H _a : Median Not Equal To 3.5				
Customer Type	1	2	3	
Count (N)	31	42	27	
Median	3.560	4.340	3.510	
Points Below 3.5	15	5	13	
Points Equal To 3.5	0	1	0	
Points Above 3.5	16	36	14	
P-Value (2-sided)	1.0000	0.0000	1.0000	

Hypothesis Test Example

Example: Fitness Center

- A hypothesis test is being conducted about the median age of female members of the Trim Fitness Center.

H_0 : Median Age = 34 years

H_a : Median Age $\neq$ 34 years

- In a sample of 40 female members, 25 are older than 34, 14 are younger than 34, and 1 is 34.
- Is there sufficient evidence to reject H_0 ? Assume $\alpha = .05$.



Non-Parametric Testing

AutoSave (Off) Book1 - Excel Search

File Home Insert Draw Page Layout Formulas Data Review View Help Acrobat SigmaXL

Data Manipulation Templates and Calculators Graphical Tools Statistical Tools Measurement Systems Analysis Process Capability Design of Experiments Control Charts Reliability/Weibull Analy

A1 X ✓ fx AG

	A	B	C	D	J	K	L
1	AGE						
2	35						
3	35						
4	35						
5	35						
6	35						
7	35						
8	35						
9	35						
10	35						
11	35						
12	35						
13	35						
14	35						
15	35						
16	35						
17	35						
18	35						
19	35						
20	35						
21	35						
22	35						
23	35						

Sheet1 1 Sample Sign Te

Basic Statistical Templates >

- Descriptive Statistics
- 1 Sample t-Test & Confidence Intervals
- Paired t-Test
- 2 Sample t-Test
- 2 Sample Comparison Tests
- One-Way ANOVA & Means Matrix
- Welch's ANOVA (Assume Unequal Variance)
- Two-Way ANOVA
- Equal Variance Tests >
- Correlation Matrix
- Regression >
- Chi-Square Tests >
- Chi-Square Tests - Exact >
- Nonparametric Tests >
- Nonparametric Tests - Exact >
- Power & Sample Size Calculators >
- Power & Sample Size with Worksheets >

1 Sample Sign

1 Sample Wilcoxon

2 Sample Mann-Whitney

Kruskal-Wallis

Mood's Median Test

Non-Parametric Testing

A3						
	A	B	C	D	E	F
1	1 Sample Sign Test					
2						
3	Test Information					
4	H_0 : Median = 34					
5	H_a : Median Not Equal To 34					
6						
7	Results:	Column 1				
8	Count (N)	40				
9	Median	35				
10	Points Below 34	14				
11	Points Equal To 34	1				
12	Points Above 34	25				
13	P-Value (2-sided)	0.1081				
14						
15						

SigmaXL Wilcoxon Signed – Rank Test

This test is an alternative to the 1-Sample Sign Test

Historically, our Median customer satisfaction score has been 3.5. We would like to see if this has changed, with the results grouped by customer type

Ho: Median=3.5

Ha: Median $\neq$ 3.5

$\alpha = 0.05$).

SigmaXL > Statistical Tools > Nonparametric Test > 1 Sample Wilcoxon

1 Sample Wilcoxon

Customer Record No
Order Date
Avg No. of orders per mo
Avg days Order to delivery
Loyalty - Likely to Recomm
Responsive to Calls
Ease of Communications
Staff Knowledge
Size of Customer
Major-Complaint
Product Type
Sat-Discrete

☒ **Stacked Column Format (1 Numeric Data Column & 1 Optional Group Category)**
☐ **Unstacked Column Format (1 or more Numeric Data Columns)**

Numeric Data Variable (Y) >> Overall Satisfaction

Optional Group Category (X) >> Customer Type

<< Remove

H0: Median = 3.5

Ha: Not Equal To

OK >>
Cancel
Help

SigmaXL Wilcoxon Signed – Rank Test

1 Sample Wilcoxon Test: Overall Satisfaction			
Test Information			
H_0 : Median = 3.5			
H_a : Median Not Equal To 3.5			
Customer Type	1	2	3
Count (N)	31	42	27
Count for Test	31	41	27
Median	3.56	4.34	3.51
Wilcoxon Statistic	217.50	802.50	222.00
P-Value (2-sided)	0.5566	0.0000	0.4349

Note:

The P-Values. Customer Type 2 shows a significant change (increase) in Satisfaction Median (P-Value < .05), whereas Customer Types 1 and 3 show no change (P-Value $\geq$.05).

SigmaXL Mann – Whitney Test

- We will look at comparing medians of Customer Satisfaction by Customer Type, (H_0 : Median₂ = Median₃, H_a : Median₂ $\neq$ Median₃.)
- SigmaXL > Statistical Tools > Nonparametric Tests > 2 Sample Mann-Whitney

2 Sample Mann-Whitney

Customer Record No
Order Date
Avg No. of orders per mo
Avg days Order to delivery
Loyalty - Likely to Recomm
Responsive to Calls
Ease of Communications
Staff Knowledge
Size of Customer
Major-Complaint
Product Type
Sat-Discrete

☒ **Stacked Column Format (1 Numeric Data Column & 1 Group Category Column)**
☐ **Unstacked Column Format (2 Numeric Data Columns)**

Numeric Data Variable (Y) >> Overall Satisfaction
Group Category (X) >> Customer Type
 << Remove

Ha: Not Equal To

OK >>
 Cancel
 Help

SigmaXL Mann – Whitney Test

Given the P-Value of .0008 we reject H_0 and conclude that Median Customer Satisfaction is significantly different between Customer types 2 and 3.

Select 2 items from the list (in desired order for one sided comparison).

1

>>

2
3

<< Remove

OK >>
Cancel
Help

2 Sample Mann-Whitney - Overall Satisfaction		
Test Information		
H_0 : Median Difference = 0		
H_a : Median Difference $\neq$ 0		
Customer Type	2	3
Count	42	27
Median	4.340	3.510
Mann-Whitney Statistic	1744.00	
P-Value (2-sided, adjusted for ties)	0.0008	

Mann – Whitney Test

Example: Express Deliveries

- A firm has decided to select one of two express delivery services to provide next-day deliveries to its district offices.
- To test the delivery times of the two services, the firm sends two reports to a sample of 10 district offices, with one report carried by one service and the other report carried by the second service. **Do the data on the next slide indicate a difference in the two services?**



Mann – Whitney Test

<u>District Office</u>	<u>OverNight</u>	<u>NiteFlite</u>
Seattle	32 hrs.	25 hrs.
Los Angeles	30	24
Boston	19	15
Cleveland	16	15
New York	15	13
Houston	18	15
Atlanta	14	15
St. Louis	10	8
Milwaukee	7	9
Denver	16	11



Mann – Whitney Test

Hypotheses

- H_0 : The delivery times of the two services are the same; neither offers faster service than the other.
- H_a : Delivery times differ between the two services; recommend the one with the smaller times.

Mann – Whitney Test

Excel spreadsheet showing data for District Office and flight types (Overnight, NightFlight) across various cities. A 2 Sample Mann-Whitney dialog box is open, showing the Unstacked Column Format selected, with variables Overnight and NightFlight listed. The Ha: dropdown is set to Not Equal To.

District Office	Overnight	NightFlight
Seattle	32	25
Los Angeles	30	24
Boston	19	15
Cleveland	16	15
New York	15	13
Houston	18	15
Atlanta	14	15
St. Louis	10	8
Milwaukee	7	9
Denver	16	11

2 Sample Mann-Whitney

☐ Stacked Column Format (1 Numeric Data Column & 1 Group Category Column)

☒ Unstacked Column Format (2 Numeric Data Columns)

Numeric Data Variables (Y) >>

Overnight
NightFlight

<< Remove

Ha: Not Equal To

OK >>
Cancel
Help

Mann – Whitney Test

A3				
Test Information				
	A	B	C	D
1	2 Sample Mann-Whitney			
2				
3	Test Information			
4	H ₀ : Median Difference = 0			
5	H _a : Median Difference ≠ 0			
6				
7	Results:	Overnight	NightFlight	
8	Count	10	10	
9	Median	16	15	
10				
11	Mann-Whitney Statistic	119.00		
12	P-Value (2-sided, adjusted for ties)	0.3036		
13				
14				
15				
16				
17				
18				

Mann – Whitney Test

Conclusion

- Do NOT Reject H_0 .
- The p -value for this two-tail test is 0.3036
- There is NOT enough evidence in the sample to conclude that a difference exists in the delivery times by the two services.



Mann – Whitney Test

Example: Westin Freezers

- Manufacturer labels indicate the annual energy cost associated with operating home appliances such as freezers.
- The energy costs for a sample of 10 Westin freezers and a sample of 10 Easton Freezers are shown on the next slide.
- Does the data indicate, using $\alpha = .05$, that a difference exists in the annual energy costs for the two brands of freezers?



Mann – Whitney Test

Westin Freezers

\$55.10

54.50

53.20

53.00

55.50

54.90

55.80

54.00

54.20

55.20

Easton Freezers

\$56.10

54.70

54.40

55.40

54.10

56.00

55.50

55.00

54.30

57.00

Mann – Whitney Test

Hypotheses

- H_0 : Annual energy costs for Westin freezers and Easton freezers are the same.
- H_a : Annual energy costs differ for the two brands of freezers.



Mann – Whitney Test

Conclusion

Do not reject H_0 . The p -value $> \alpha$. There is insufficient evidence in the sample data to conclude that there is a difference in the annual energy cost associated with the two brands of freezers.



Kruskal – Wallis Test

- The Kruskal Wallis test is used for cases of three or more populations.

H_0 : All populations are identical

H_a : Not all populations are identical

- The Kruskal-Wallis test can be used with ordinal data as well as with interval or ratio data.
- Also, the Kruskal-Wallis test does not require the assumption of normally distributed populations.

SigmaXL Kruskal – Wallis Test

SigmaXL > Statistical Tools > Nonparametric Tests > Kruskal-Wallis

Kruskal-Wallis Nonparametric ANOVA

☒ Stacked Column Format (1 Numeric Data Column & 1 Group Category Column)
☐ Unstacked Column Format (2 or more Numeric Data Columns)

Numeric Data Variable (Y) >> Overall Satisfaction

Group Category (X) >> Customer Type

<< Remove

OK >>
Cancel
Help

Customer Record No
Order Date
Avg No. of orders per m
Avg days Order to deliv
Loyalty - Likely to Recor
Responsive to Calls
Ease of Communication
Staff Knowledge
Size of Customer
Major-Complaint
Product Type
Sat-Discrete

SigmaXL Kruskal – Wallis Test

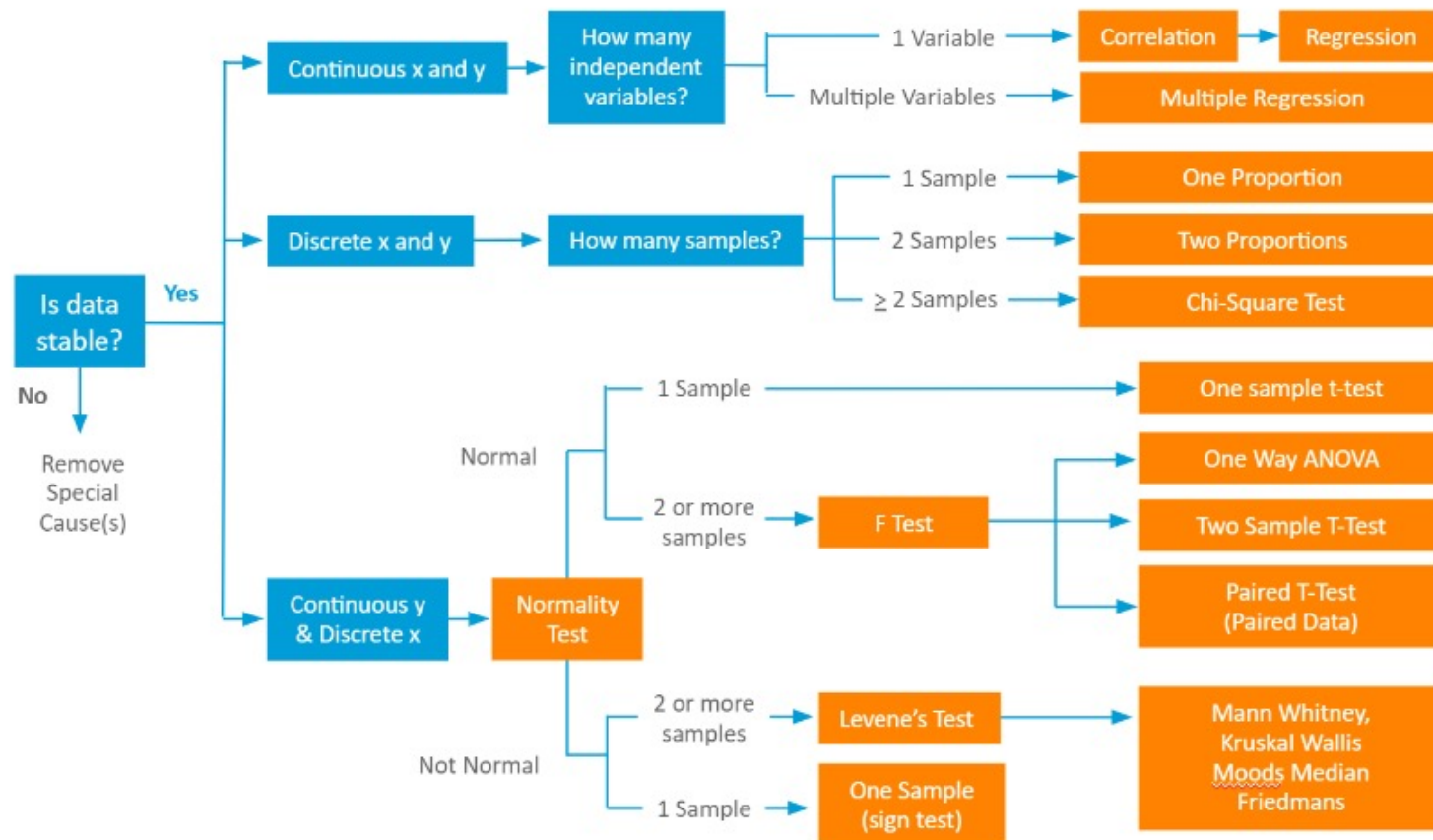
The P-Value of 0.000023 (2.3 e-5) tells us that we reject H0.
At least one pairwise set of medians are not equal.

Kruskal-Wallis Nonparametric ANOVA: Overall Satisfaction			
Test Information			
H ₀ : Median 1 = Median 2 = ... = Median k			
H _a : At least one pair Median i ≠ Median j			
Customer Type	1	2	3
Count (N)	31	42	27
Median	3.56	4.34	3.51
UC Median (2-sided, 95%)	3.936	4.518	4.023
LC Median (2-sided, 95%)	2.954	4.095	3.289
Z	-3.339	4.529	-1.557
Kruskal-Wallis Statistic (H)	21.360		
DF	2		
P-Value (2-sided, adjusted for ties)	0.000023		

Hypothesis Testing

Attribute Data

Hypothesis Testing Roadmap



Sample Size and Types of Data

For *Continuous Data*:

Capability Analysis – a minimum of approx 30 samples

Hypothesis Testing – Approx 30 samples but depends on the practical difference to be detected and the inherent variation in the process as well as the statistical confidence you wish to have.

For *Attribute Data*:

Capability Analysis – More than 100 samples

Hypothesis Testing – More than 100 samples ... but depends on practical difference to be detected as well as the statistical confidence you wish to have.

Proportion vs a Target

This test is used to determine if the process proportion (p) equals some desired value, p_0 .

The hypotheses:

$$H_o: p = p_0$$

$$H_a: p \neq p_0$$

NOTE:

Be careful not to confuse this 'p' for 'proportion' with the 'p' for 'probability (p-value) .

LOOK AT THE CONTEXT OF HOW IT IS BEING USED !

1 Proportion vs a Target

1. Shipping accuracy has a target of 99%; determine if the current process is on target.
2. Hypotheses:
 - $H_0: p = 0.99$
 - $H_a: p \neq 0.99$
3. One sample proportion test
 - Choose $\alpha = 5\%$
4. Run the test on the data below:
 - Number of items shipped = 500
 - Number of items on time = 480

1 Proportion vs a Target

SigmaXL > Templates and Calculators > Basic Statistical Templates > 1 Proportion Test...

SigmaXL 1 Proportion Test and Confidence Interval		
Sample Data (user inputs):		
Number of Events	x	480
Sample Size	n	500
Null Hypothesis (hypothesized proportion)	H_0 : Proportion =	0.5
Alternative Hypothesis	H_a : Proportion	Not Equal To
Confidence Level (enter .95 for 95%)	$100 \cdot (1 - \alpha)\%$	95.0%
Hypothesis Test Method		Binomial Exact
Confidence Interval Method		Exact (Clopper-Pearson Beta)
Results:		
Sample proportion (x/n)		0.9600
alpha		0.0500
npq (npq should be ≥ 5 for normal approximation; $q = 1 - p$)		19.2000
Z-statistic (normal)		20.5718
Binomial exact probability P-Value (2-sided)		0.0000
Upper Confidence Limit (2-sided)		0.9754
Lower Confidence Limit (2-sided)		0.9389

5. **Statistical Conclusion:** Reject the null hypothesis because the hypothesized Mean is not within the confidence interval.
6. **Practical Conclusion:** We are not performing to the accuracy target of 99%.

2 Proportion Test

1. The sponsor of a process improvement project for an insurance claims process wants to know with 95% confidence that the proportion of accurate claims forms after the improvement is different than the proportion of accurate claims forms before the improvement.
2. **Hypotheses:**
 - $H_o: p_1 - p_2 = 0.0$
 - $H_a: p_1 - p_2 \neq 0.0$
3. **Two sample proportion test**
 - Choose $\alpha = 5\%$
4. **Run the test**
5. **Statistical Conclusion**
6. **Practical Conclusion**

Comparing Two Proportions

Statistical Conclusion:

Reject the null

Practical Conclusion: You have achieved a significant difference in accuracy due to the improvement.

 2 Proportions Test and Confidence Interval			
Sample Data (user inputs):		Sample 1	Sample 2
Number of Events	x	510	212
Sample Size	n	600	225
Null Hypothesis (hypothesized difference)	$H_0: P_1 - P_2 =$	0	
Alternative Hypothesis	$H_a: P_1 - P_2$	Not Equal To	
Confidence Level (enter .95 for 95%)	$100 \cdot (1 - \alpha) \%$	95.0%	
Hypothesis Test Method		Fisher's Exact	
Confidence Interval Method		Normal Approximation	
Results:			
Sample proportion (x/n)		0.8500	0.9422
Sample proportion difference		-0.0922	
alpha		0.0500	
Minimum expected value (should be ≥ 5 for normal approximation)		28.0909	
Fisher's Exact probability P-Value (2-sided)		0.0002	
Upper Confidence Limit (2-sided)		-0.0504	
Lower Confidence Limit (2-sided)		-0.1340	

Exercise

Exercise objective: To practice solving a problem presented using the appropriate Hypothesis Test.

Boris and Igor tend to make a lot of mistakes writing requisitions.

	# Req's	# Wrong
Boris	356	47
Igor	571	99

1. Who is worse?
2. Is the sample size large enough?

Contingency Tables

Contingency Tables are used to simultaneously compare more than two sample proportions with each other.

It is called a **Contingency Table** because we are testing if the proportion is contingent upon, or dependent upon, the factor used to subgroup the data.

Some examples for use include:

- Return proportion by product line
- Claim proportion by customer
- Defect proportion by manufacturing line

Contingency Tables

The null hypothesis is that the population proportions of each group are the same.

- **H_o: $p_1 = p_2 = p_3 = \dots = p_n$**
- **H_a: at least one p is different**

The calculation uses :

$$\chi^2 \text{ value} = \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

Statisticians have shown the following statistic forms a **Chi-square distribution** (*See Chi-Square table*)

Contingency Table Example 1

You are interested in the relationship between gender and political party affiliation.

You query 100 people about their political affiliation and record the number of males (row 1) and females (row 2) for each political party.

The worksheet data appears as follows:

1. Open the worksheet EXH_TABL.xlsx.
2. Choose **SigmaXL > Statistical Tools > Chi-Square Tests >**
3. **Chi-Square Test and Association – Two-Way Table Data**
4. Check “Nominal Categories”.
5. Click “Next >>”

Contingency Table Example 1

Statistical Tools > Chi-SquareTests > Chi-Square Test and Association – Two-Way Table Data

Chi-Square Test & Association - Two-Way Table Data

Please select your data

\$A\$1:\$D\$3

Note:
1. Two-Way (pivot) table must be based on a single Y or count variable.
2. Selection should include only one row and only one column label.
3. Selection should exclude Grand Totals.
See Help for example.

Advanced Tests and Measures of Association

☒ Nominal Categories
☐ Ordinal Categories

Confidence Level: 95.0

Help Cancel Next >>

	A	B	C	D
	Democrat	Republican	Other	
Male	28	18	4	
Female	22	27	1	

Contingency Table Example 1

Interpreting the results

Both p-values are greater than 0.05.

Thus, there is no evidence of an association between gender and political party affiliation.

Of the 6 cells, 2 have expected counts less than five (33%).

Therefore, even if you had a significant p-value for these data, you should interpret the results with caution. To be more confident of the results, repeat the test, omitting the Other category.

Pearson Chi-Square = 4.320, DF = 2, P-Value = 0.115

Likelihood Ratio Chi-Square = 4.461, DF = 2, P-Value = 0.107

*** NOTE * 2 cells with expected counts less than 5**

Contingency Table Example 2

1. Zurich, London and Paris are distribution centres for and you suspect one of them has a lower rate of late deliveries than the others during the past 10 days.

2. $H_0: p_{\text{zurich}} = p_{\text{London}} = p_{\text{Paris}}$

H_a : at least one p is different

3. Use Contingency Table since there are 3 proportions.

	Zurich	London	Paris	
Late Deliveries in past 10 days	50	55	100	
Total Deliveries in past 10 days	2000	3000	5000	

Contingency Table Example 2

Using SigmaXL

Type the delivery data into SigmaXL as shown
or use the data supplied in the BB data:

F	G	H	I
	Zurich	London	Paris
Late Deliveries in past 10 days	50	55	100
Total Deliveries in past 10 days	2000	3000	5000

Contingency Table Example 2

Statistical Tools > Chi-SquareTests > Chi-Square Test and Association – Two-Way Table Data

Chi-Square Test & Association - Two-Way Table Data

Please select your data

Note:

1. Two-Way (pivot) table must be based on a single Y or count variable.
2. Selection should include only one row and only one column label.
3. Selection should exclude Grand Totals.

See Help for example.

Advanced Tests and Measures of Association

☒ Nominal Categories
☐ Ordinal Categories

Confidence Level:

Help
Cancel
Next >>

Contingency Table Example 2

Chi-Square Two-Way Table Statistics			
Observed Counts	Zurich	London	Paris
Late Deliveries in past 10 days	50	55	100
Total Deliveries in past 10 days	2000	3000	5000
Expected Counts	Zurich	London	Paris
Late Deliveries in past 10 days	41.181	61.369	102.45
Total Deliveries in past 10 days	2008.8	2993.6	4997.6

Chi-Square	2.662		
DF	2		
P-Value 	0.2642		
Additional Chi-Square Tests			
Test	Chi-Square	DF	P-Value
Likelihood Ratio	2.5645	2	0.2774
McNemar-Bowker Symmetry (Square Table)			

5. Statistical Conclusion: P-value is > 0.05 so Stay with the null hypothesis.

6. Practical Conclusion: There is insufficient evidence to say that late delivery rate is dependent on (it is NOT contingent on) Distribution Centre location.

Summary

- Contingency Tables are used to test for association (or dependency) between different classifications based on comparing **proportions** in each classification.
- The null hypothesis is that the classifications are independent.
- A Chi-square Test is used to test for dependency

Summary

At this point you should be able to:

- Calculate and explain test for Non-Parametric Tests
- Calculate and explain tests for Proportions
- Calculate and explain Contingency Tests
- Explain and execute a Chi-squared Test



Advanced Design of

Experiments

Types of DOE Experimental Methods

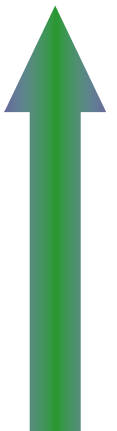
- ***Fractional Factorial***
 - 6 -15 input variables
- ***Full Factorial***
 - 2-6 input variables
- ***Response Surface Method (RSM) / Method of Steepest Ascent***
 - 2-3 input variables

RESPONSE SURFACE METHODS
Steepest Ascent Design

Full Factorial

Fractional Factorials

KNOWLEDGE

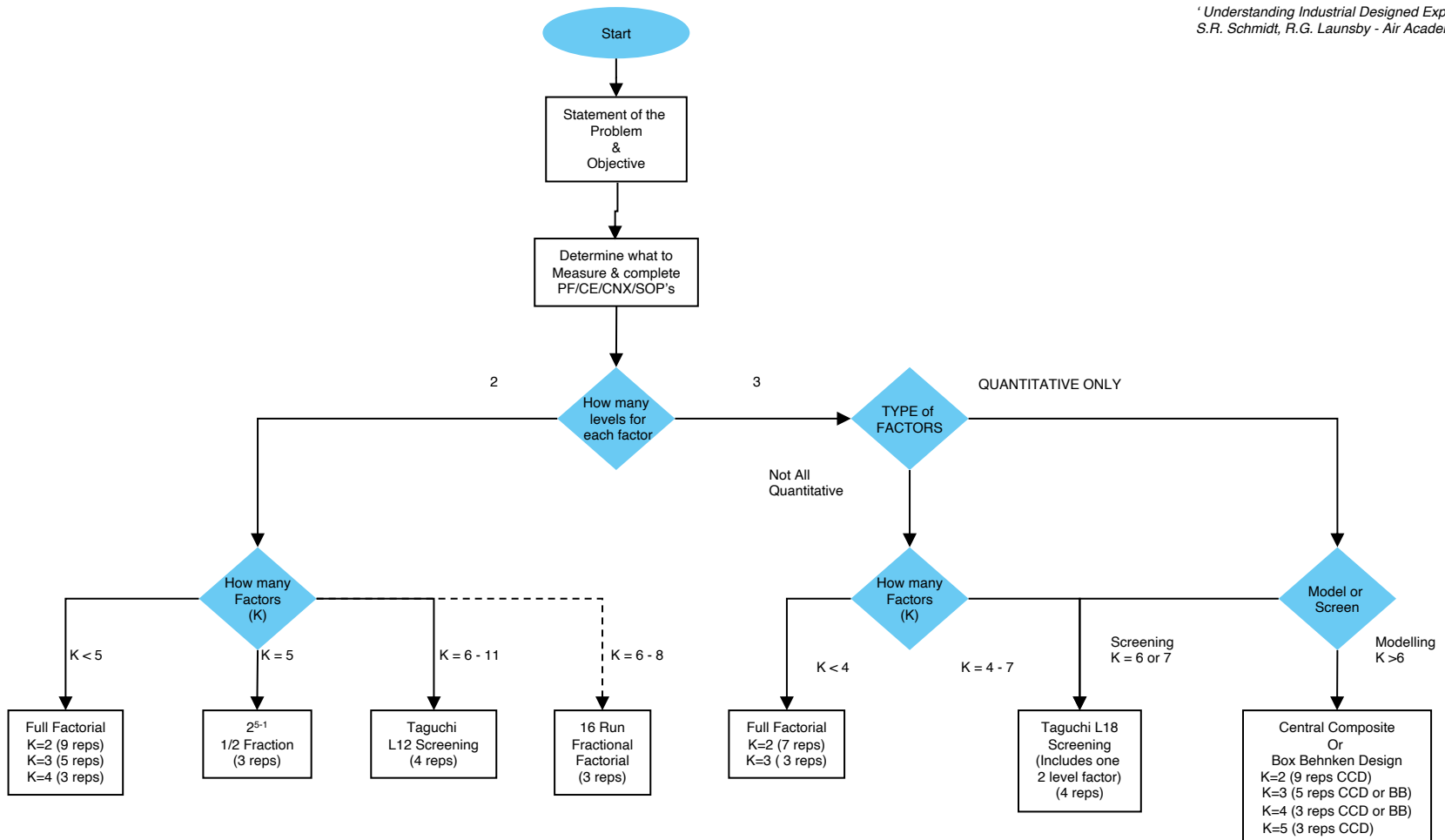


Design Types

- **FULL Factorials**
- **Fractional Factorials**
 - Taguchi Othogonal Designs
 - Plackett-Burman
 - Box – Behnken designs
 - Central composite designs

Diagram

'Understanding Industrial Designed Experiments'
 S.R. Schmidt, R.G. Launsby - Air Academy Press



Taguchi Designs – Orthogonal arrays (DA)

L4 (2^2)

L8 (2^7)

L12 (2^{11})

L16 (2^{15})

L9 (3^4)

L27 (3^{13})

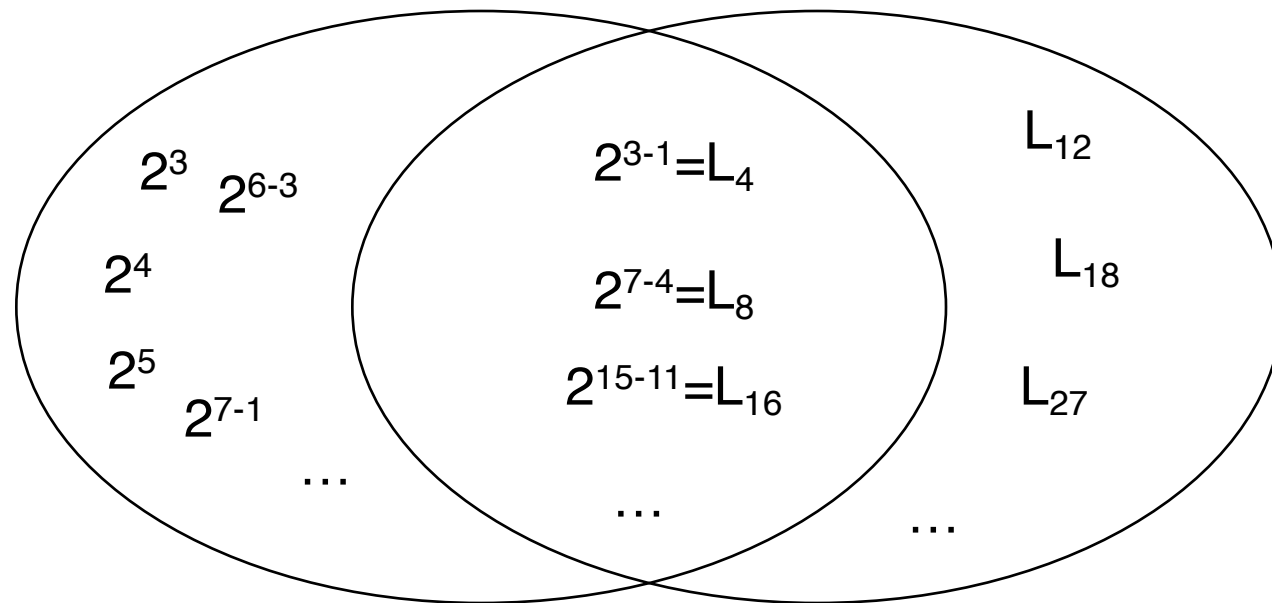
Run	Columns										
	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	2	2	2	2	2	2
3	1	1	2	2	2	1	1	1	2	2	2
4	1	2	1	2	2	1	2	2	1	1	2
5	1	2	2	1	2	2	1	2	1	2	1
6	1	2	2	2	1	2	2	1	2	1	1
7	2	1	2	2	1	1	2	2	1	2	1
8	2	1	2	1	2	2	2	1	1	1	2
9	2	1	1	2	2	2	1	2	2	1	1
10	2	2	2	1	1	1	1	2	2	1	2
11	2	2	1	2	1	2	1	1	1	2	2
12	2	2	1	1	2	1	2	1	2	2	1

http://reliawiki.org/index.php/Taguchi_Orthogonal_Arrays

Orthogonal Designs

“Classical”
(2-level Factorials)

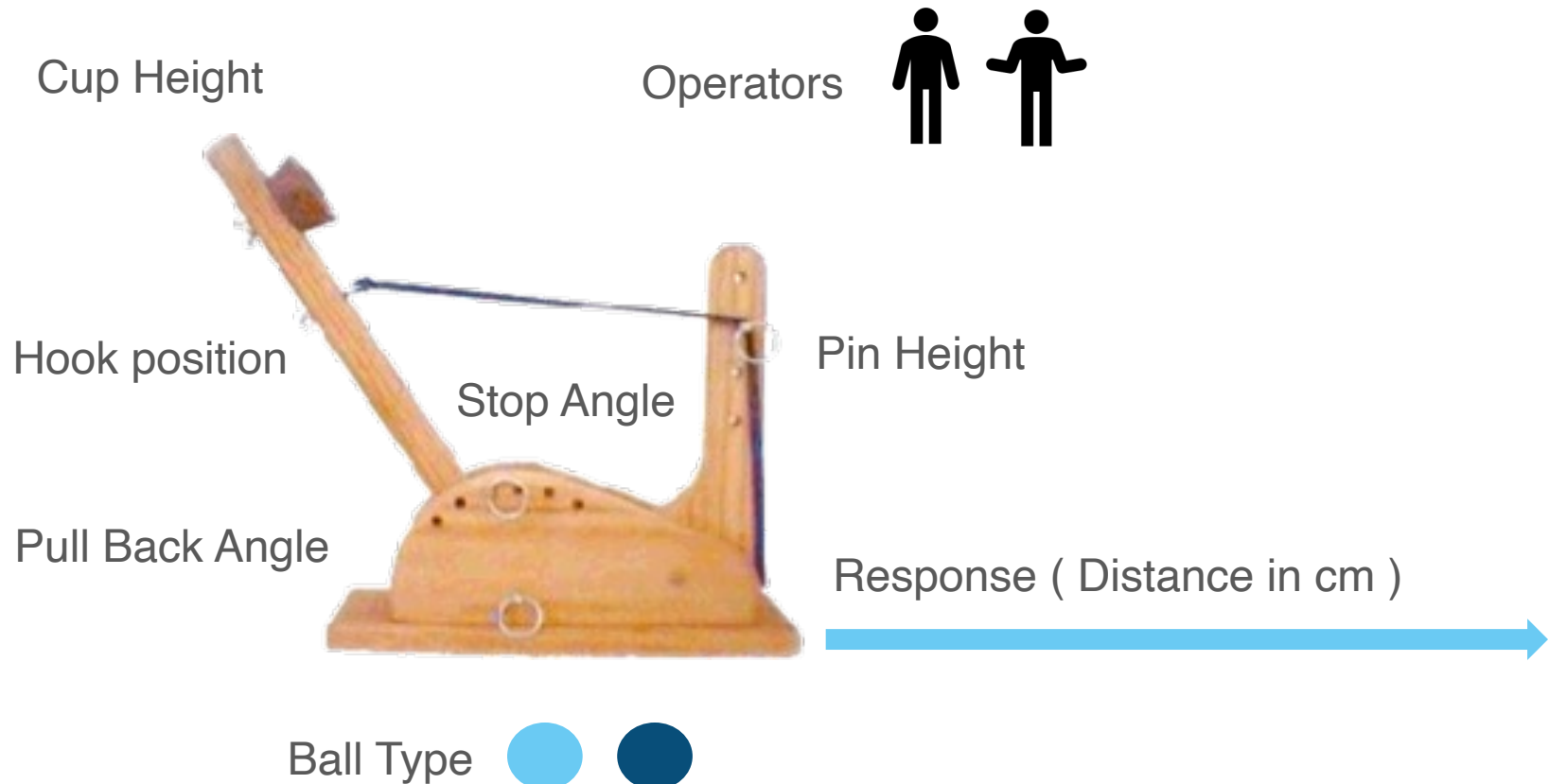
“Taguchi”



8 – Step DOE Methodology

1. Define the Practical Problem
2. Select the Output (response) Variable
3. Select the Input Factors (predictor) Variables
4. Choose the Levels for the Input Factors Variables
5. Select the Experimental Design and enter factors and levels
6. Execute the Experiment and collect data
7. Analyze the Data from the designed experiment and draw Statistical Conclusions
8. Draw Practical Conclusion and Solutions

Experiment on Catapult



Exercise Taguchi

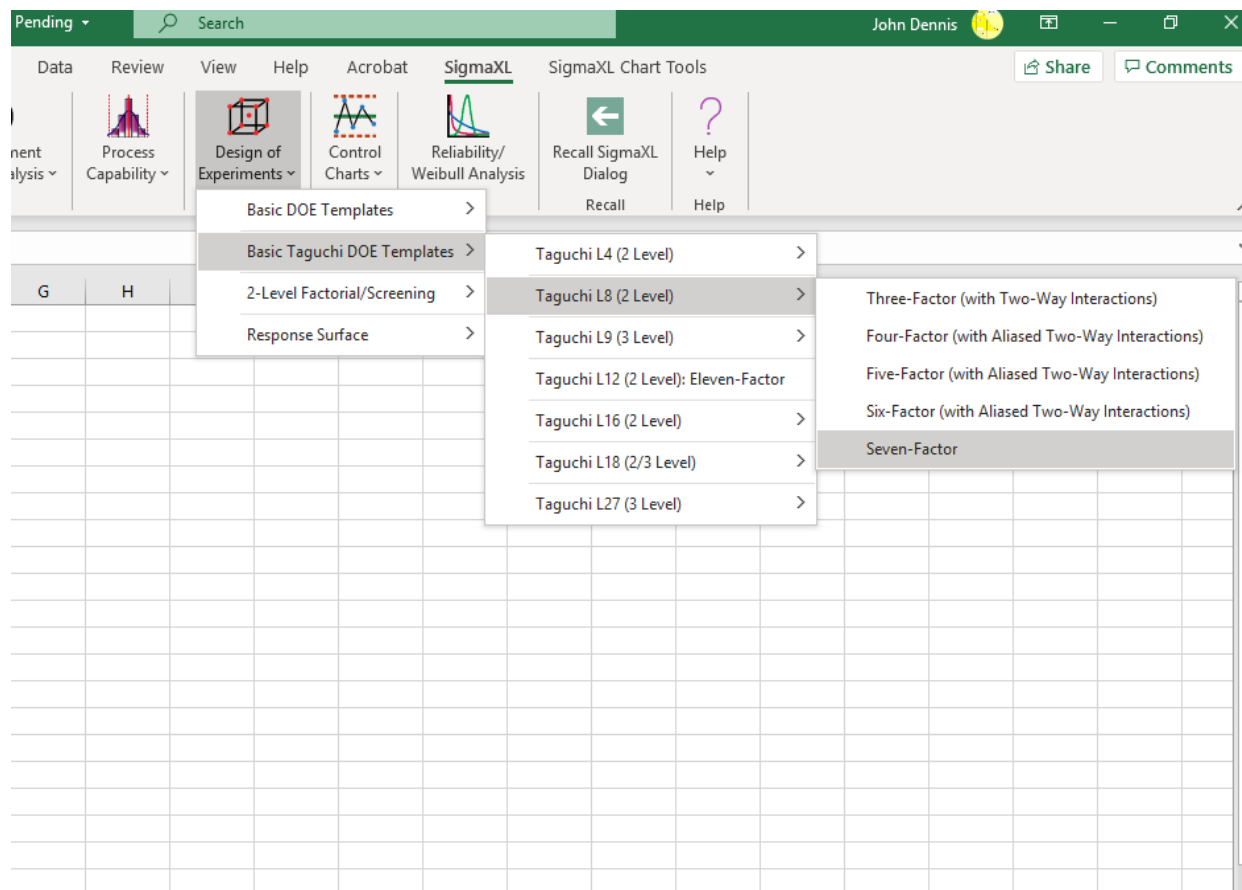
Conduct a Taguchi L_8 Screening design using the following 7 factors and levels:

FACTORS

LEVELS

	<u>Low(-)</u>	<u>High (+)</u>
A: Start Angle	160 ⁰	180 ⁰
B: Stop Angle	2	4
C: Hook	2	4
D: Arm Length	4	5
E: Rubber Band Position	1	3
F: Ball Type	Ball 1	Ball 2
G: Operator	Op 1	Op 2

Taguchi L8 Design



Add the Factors

1. Select Signal-to-Noise Ratio to Nominal is Best MSD with Target
(**MSD = Mean Square Deviation and is used when Variance is measured relative to the Target and not the Average**)
2. Select Standard Deviation Response to use StDev(y)
3. Enter Factor Names and Factor Levels
4. Set the **TARGET** To whatever you want to HIT (50 cm for example)

Taguchi L8

Signal-to-Noise Ratio:	SN: Nominal is Best (MSD with Target)	SN Formula: $-10 \cdot \log_{10}(\text{Sum}((Y-T)^2/n))$
Standard Deviation Response:	StDev (Y)	

MSD Target:	50
-------------	----

Factor	Factor Name	Level 1	Level 2
A	Hook	Low	High
B	Arm Length	Short	Long
C	Start Angle	150	177
D	Stop Angle	Hole 4	Hole 5
E	Operator	1	2
F	Ball Type	1	2
G	Pin Height	2	4

Predicted Output for Y:

Factor Name	Enter Actual Factor Setting - uncoded	Factor setting coded	$\hat{Y}$	$\hat{S}$	SN: Nominal is Best (MSD with Target)
Hook	1	FALSE	#VALUE!	#VALUE!	#VALUE!
Arm Length	1	FALSE			
Start Angle	1	FALSE			
Stop Angle	1	FALSE			
Operator	1	1			
Ball Type	1	1			
Pin Height	1	FALSE			

Inner Array:

Outer Array:

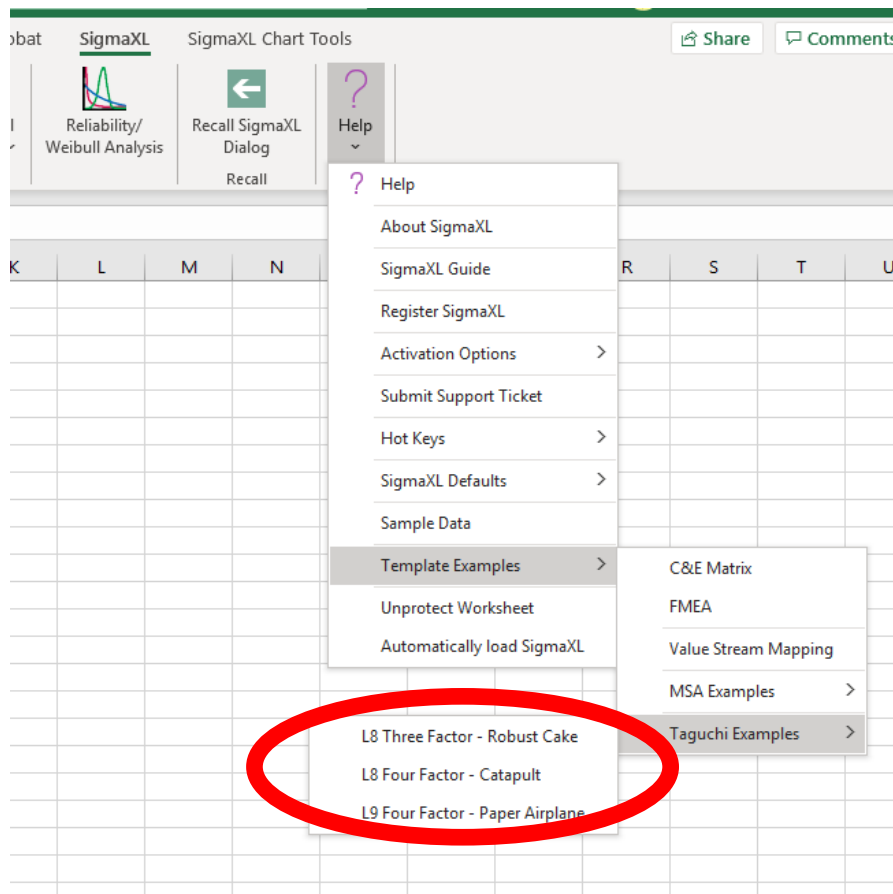
Std. Order	Run No.	Hook	Arm Length	Start Angle	Stop Angle	Operator	Ball Type	Pin Height	Yrep1	Yrep2	Yrep3
1	1	Low	Short	150	Hole 4	1	1	2			
2	2	Low	Short	150	Hole 5	2	2	4			
3	3	Low	Long	177	Hole 4	1	2	4			
4	4	Low	Long	177	Hole 5	2	1	2			
5	5	High	Short	177	Hole 4	2	1	4			
6	6	High	Short	177	Hole 5	1	2	2			

Enter Responses (Distances Travelled)

Outer Array:

[illegible]

Example



**Here is one
with 4 Factors
we already
produced !!**

Taguchi L8

C12

✕

✓

f_x

Hook

Recall

Help

	A	B	C	D	E	F	G	H	I	J	K	L	M
1	Taguchi L8 (2 Level) Orthogonal Array, Four-Factor (with Aliased Two-Way Interactions)												
2													
3	Title:	Catapult Experiment adapted from "THE CATAPULT PROBLEM: ENHANCED ENGINEERING MODELING USING EXPERIMENTAL DESIGN", Schubert et. al.											
4	Date:	Quality Engineering, 4(4), 463-473 (1992)											
5	Name of Experimenter:												
6	Response:	Distance in Inches with Target = 50"											
7													
8	Signal-to-Noise Ratio:	SN: Nominal is Best (MSD with Target)	SN Formula: $-10 \cdot \log_{10}(\text{Sum}((Y-T)^2/n))$				MSD Target:		50				
9	Standard Deviation Response:	StDev (Y)											
10													
11	Factor	Factor Name	Level 1	Level 2									
12	A	Hook	Bottom	Top									
13	B	Arm Length	Short	Long									
14	C	Start Angle	150	177									
15	D	Stop Angle	Hole 4	Hole 5									
16													
17	Aliased Interaction 1	Start Angle*Stop Angle											
18	Aliased Interaction 2	Arm Length*Stop Angle											
19	Aliased Interaction 3	Hook*Stop Angle											
20													
21	Inner Array:												
22													

Predicted Output for Y:

Factor Name	Enter Actual Factor Setting - Uncoded	Factor Setting Coded	$\hat{Y}$	$\hat{S}$	SN: Nominal is Best (MSD with Target)
Hook	Bottom	1	52.86666667	0.850490055	-9.395192526
Arm Length	Long	2			
Start Angle	177	2			
Stop Angle	Hole 4	1			

Chart Area

Taguchi L8

Aliased Interaction 1	Start Angle*Stop Angle
Aliased Interaction 2	Arm Length*Stop Angle
Aliased Interaction 3	Hook*Stop Angle

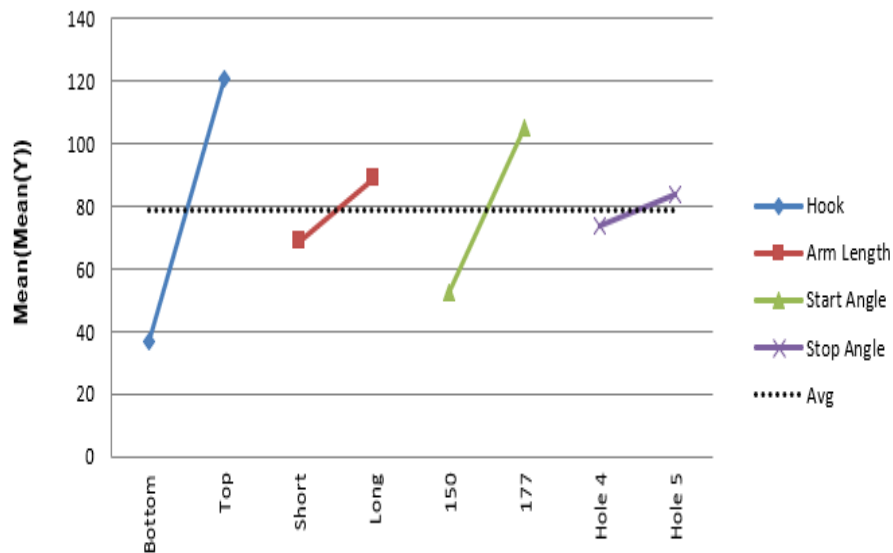
Inner Array:

Outer Array:

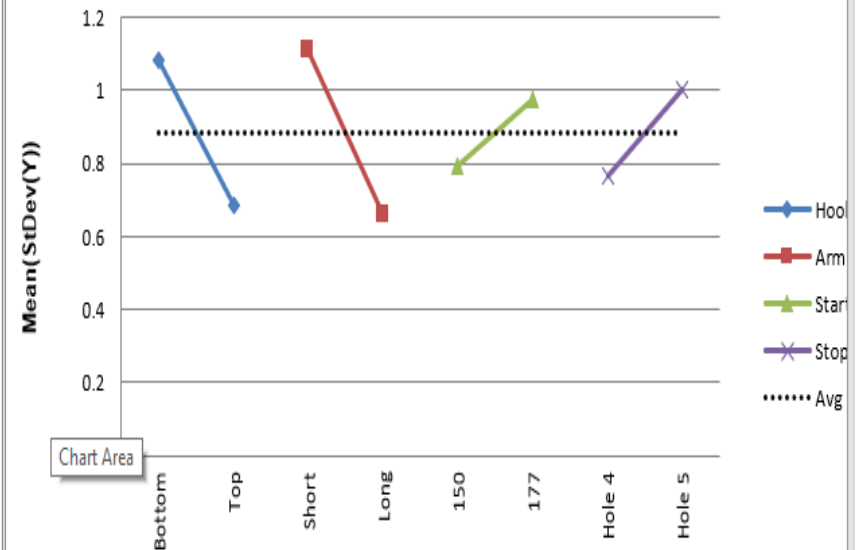
Std. Order	Run No.	Hook	Arm Length	Start Angle	Stop Angle	Yrep1	Yrep2	Yrep3	Yrep4	Yrep5	Yrep6
1	1	Bottom	Short	150	Hole 4	28	27.1	26.2			
2	2	Bottom	Short	177	Hole 5	46.3	43.5	46.5			
3	3	Bottom	Long	150	Hole 5	21.9	21	20.1			
4	4	Bottom	Long	177	Hole 4	52.9	53.7	52			
5	5	Top	Short	150	Hole 5	75	73.1	74.3			
6	6	Top	Short	177	Hole 4	127.7	126.9	128.7			
7	7	Top	Long	150	Hole 4	86.2	86.5	87			
8	8	Top	Long	177	Hole 5	195	195.7	195.7			

Analyze Main Effect Plots

Main Effects Plot for Means



Main Effects Plot for StDevs



Choose Best Setting for Best Signal and MSD to Target

MSD Target: 50

Predicted Output for Y:

Factor Name	Enter Actual Factor Setting - Uncoded	Factor Setting Coded	$\hat{Y}$	$\hat{S}$	SN: Nominal is Best (MSD with Target)
Hook	Bottom	1	52.86666667	0.850490055	-9.395192526
Arm Length	Long	2			
Start Angle	177	2			
Stop Angle	Hole 4	1			

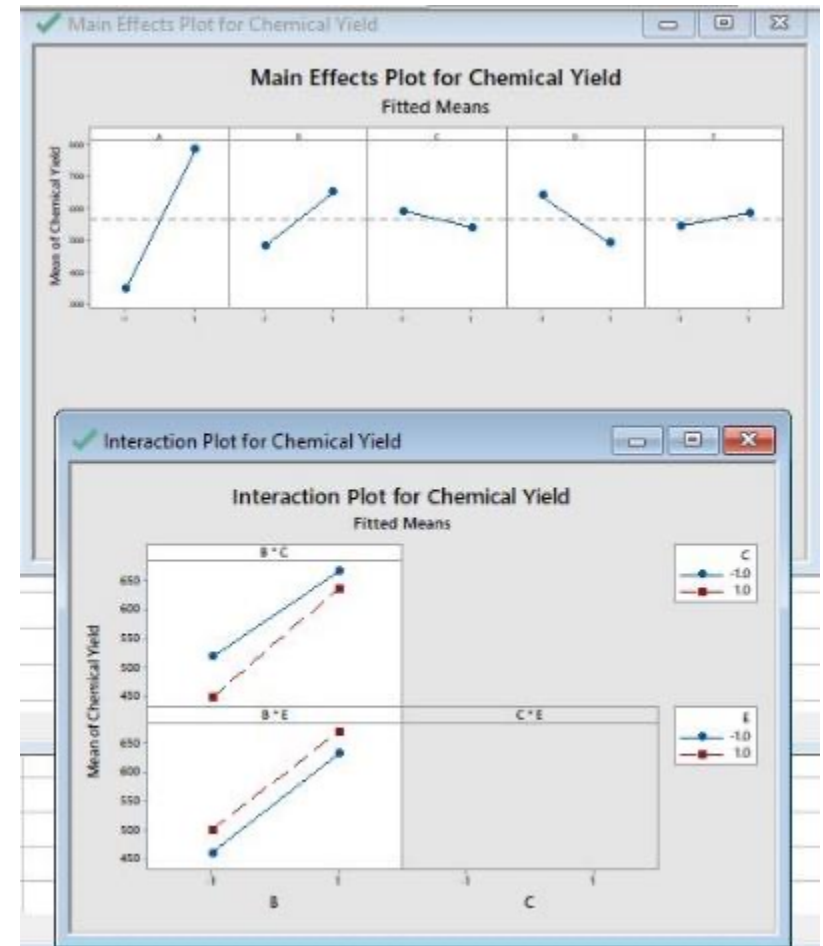
Run the Screening Experiment – Analyze Results

1. Which input Factor has the biggest Main Effect on the response ?
2. Which input Factor has the least Main Effect on the response ?
3. Which input Factors have the largest Interaction Effects ?
4. What are the 'BEST SETTINGS' you recommend to hit 50 cm Target with the least variation ?

Fractional Factorial Exercise

Main Effects and Interaction Plots

Can you interpret the Main Effects Plots and Interaction Effects Plots ?



Response Surface Methods

Response Surface Methods (RSM) & Method of Steepest Ascent (or Descent)

RSM

- The most complex and detailed methods used in DOE.
- Produces a more detailed mathematical model of a process. It does not assume that the model is linear outside of the original inference space.
- Uses a series of Fractional Factorial experiments which on each iteration get closer and closer to developing a complete 'Response Surface' to identify the optimum settings for a process.
- Due to its complexity, time and cost it is usually only used on 2 to 3 factors.

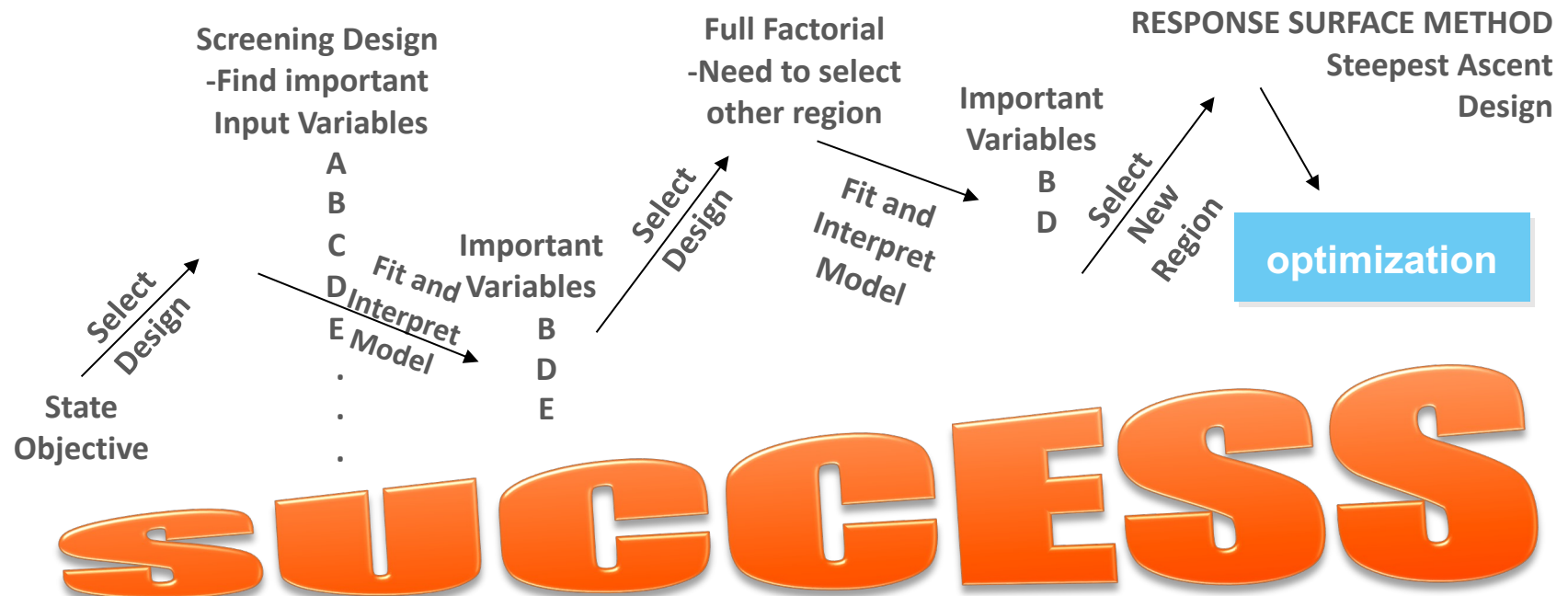
RSM and Inference Space

- Response surface method (RSM) use DOE Fractional Factorial experiments to identify the Boundaries of the “Inference Space (region)” where the Optimal Value exists for the process may exist.
- Inside the Inference Space is the region that the Fractional Factorial DOE has information about. This region may or may not hold the True optimal value for a process.
- Outside of the Inference Space is the region we do not have data for but which we want to examine because it may be where the optimization values exist.

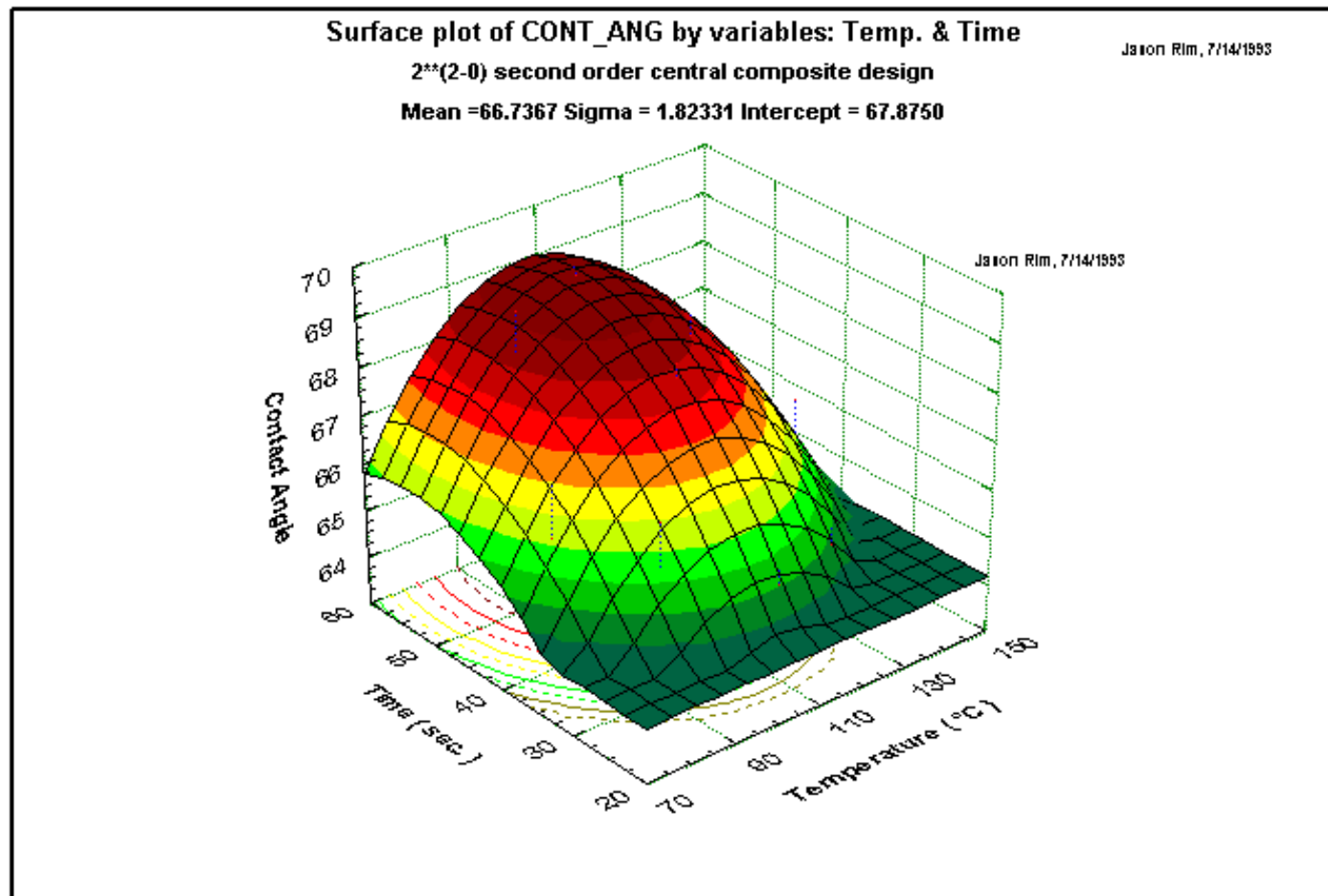
Iterative Nature of DOE

DOE is an iterative process where the approach taken depends on the information already known and how much detail (resolution) we require.

RSM and Steepest Ascent increases the knowledge about our process further and is used in conjunction with factorial experiments.



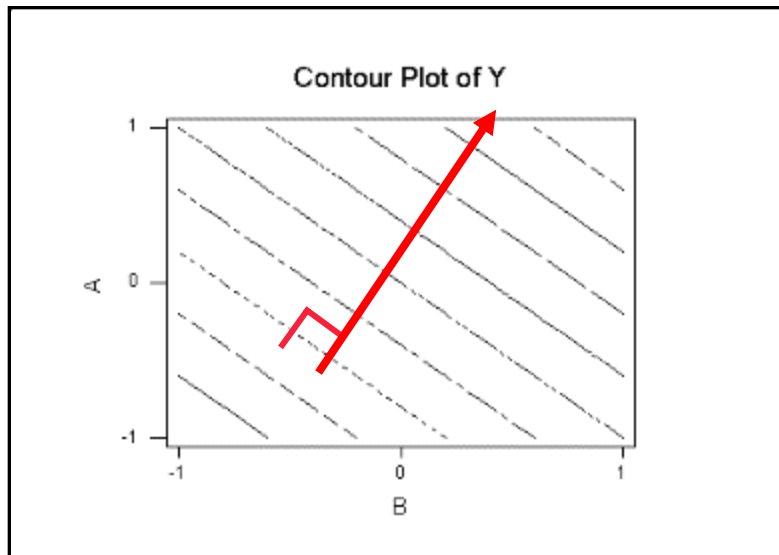
RSM Contour Plot Example



RSM / Method of Steepest Ascent

The RSM method of Steepest Ascent guides you toward an optimal target **outside the original inference space.**

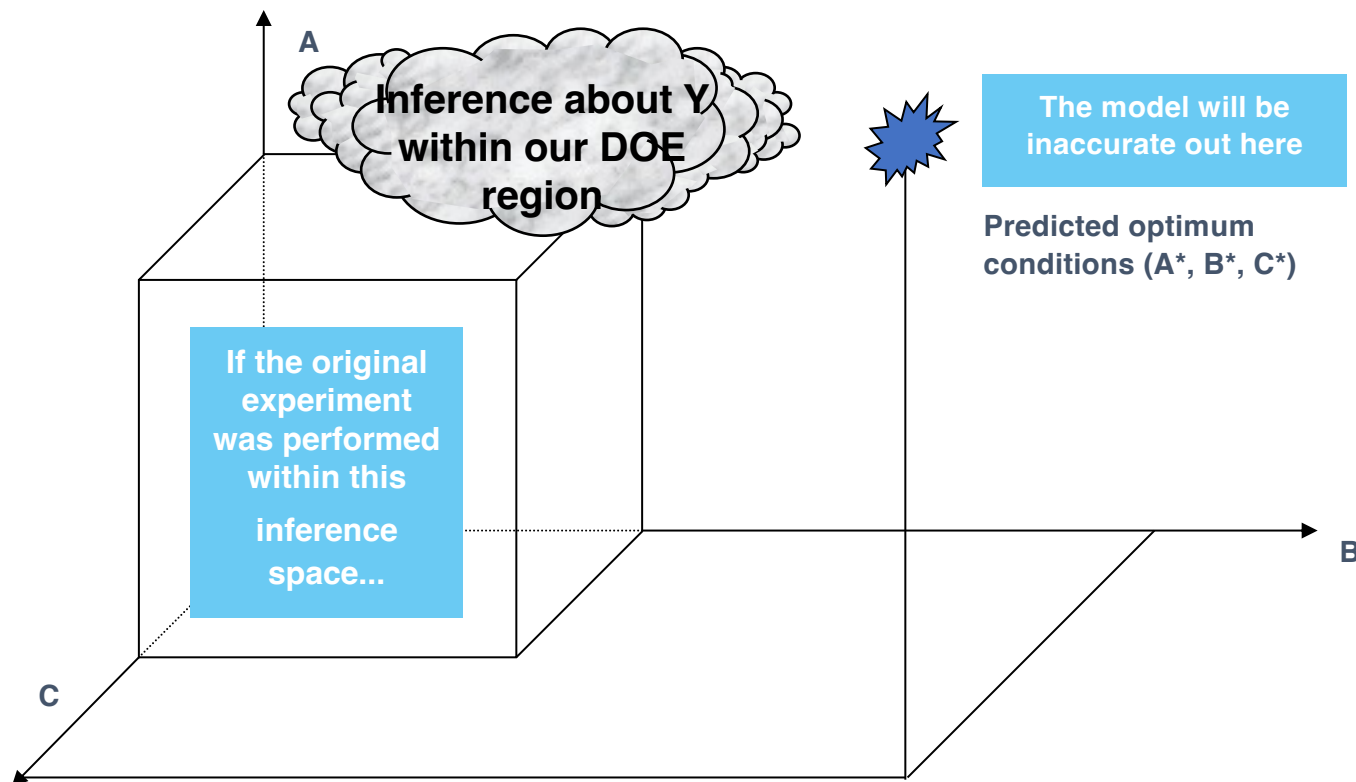
The method takes the most economical or shortest route towards the target by staying on the path of steepest ascent. In this way we reach the optimization with the fewest number of experiments (less cost / les time).



The direction of
Steepest Ascent is at a
right angle to the lines
on a Contour Plot

Inference Space

How can the Prediction from the original DOE experiments miss the mark?



Further Help With DOE and RSM

YouTube

Response surface methods to
optimise production of Popcorn

<https://www.youtube.com/watch?v=CFoj2mEVWvA>



Binomial and Poisson

Distribution

Binomial Probability Distribution

- A fixed number of observations (trials), n
 - e.g., 15 tosses of a coin; 20 patients; 1000 people surveyed
- A binary random variable
 - e.g., head or tail in each toss of a coin; defective or not defective light bulb
 - Generally called “success” and “failure”
 - Probability of success is p , probability of failure is $1 - p$
- Constant probability for each observation (p)
 - e.g., Probability of getting a tail is the same each time we toss the coin

Uses for Binomial Distribution

Binomial is used for modelling the probability of a number of chosen outcomes in a fixed number of trials when there are only two possible outcomes:

This occurs when we examine a sample from a large population of 'units' for the presence of a characteristic; each unit either has the characteristic or it doesn't.

Examples:

- Estimating the probabilities of an outcome in any set of success or failure trials
- Using sampling for attributes (acceptance sampling)
- Determining the number of defective items in a batch size of n or the spread of that distribution
- Marketing trials— 78% of buyers approve of a product
- Election poll - 44% would vote for certain political party X

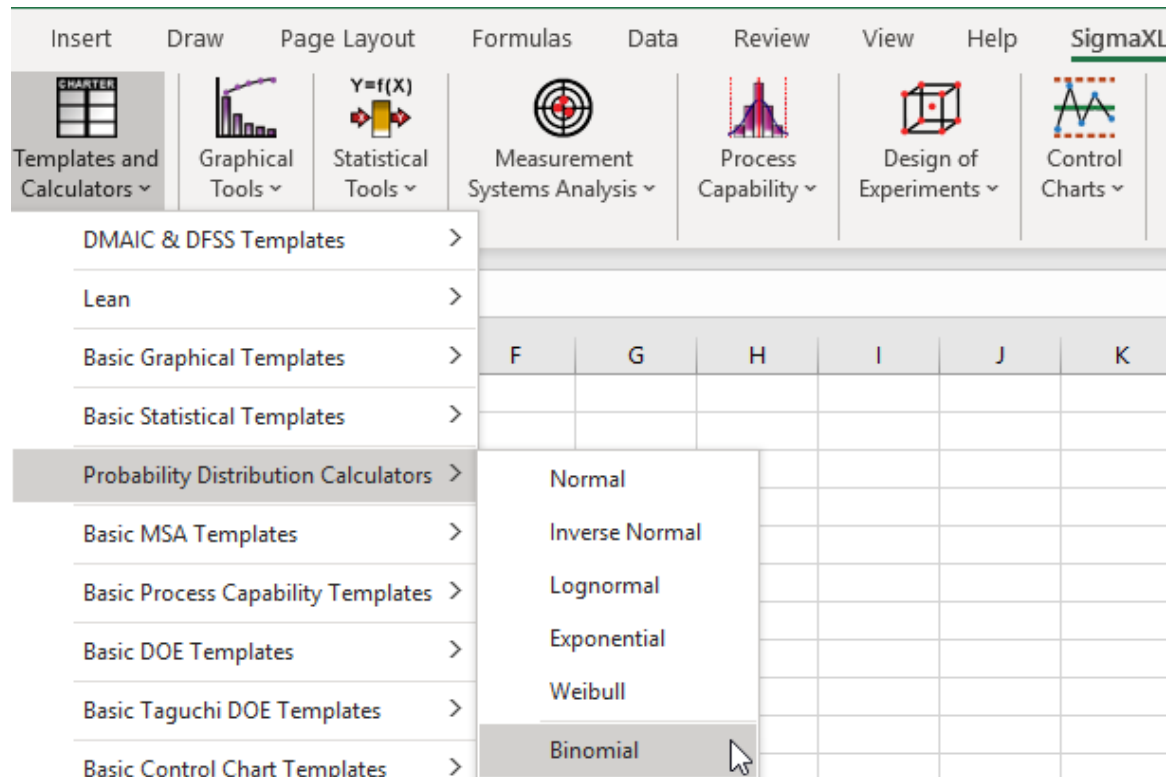
Terminology

Example: What is the chance of getting two red lights out of the three sets of traffic lights we go through to get to work?

Term	Definition	Example
n	Number of trials A trial is when we do something that will give us an outcome (flip a coin, roll a dice , traffic light colour)	3 (3 x traffic lights)
x	Number of required successful outcomes	2 (2x red lights)
p (lower case)	Probability of success (the required outcome) of each individual trial expressed as a decimal for each single trial	0.333 (One in three)
P (upper case)	The probability of getting x successes from n trials	0.19 (19% chance of getting 2 red lights out of 3 sets of lights)

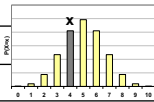
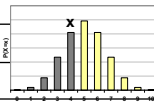
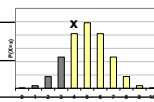
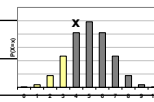
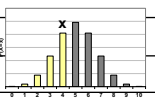
SigmaXL

Templates and Calculators → Probability Distribution Calculators → Binomial



SigmaXL

Binomial Distribution Probability Calculator		
Input the following information:		
Number of Trials	n	3
Event Probability	p	0.333
Number of Events in n Trials	x	2

Select the Probability of Interest from the following probabilities:		
	Prob(X = 2) 	0.221888889
	Prob(X ≤ 2) 	0.963073963
	Prob(X < 2) 	0.741185074
	Prob(X ≥ 2) 	0.258814926
	Prob(X > 2) 	0.036926037

n

p

x

P

Binomial Example

5 tosses of a coin. What's the probability that you land exactly 3 heads in 5 coin tosses?



Binomial Example

Binoimial Distribution Probability Calculator

Input the following information:

Number of Trials	n	5
Event Probability	p	0.5
Number of Events in n Trials	x	3

Select the Probability of Interest from the following probabilities:

equal to x	Prob(X = 3)	0.3125
less than or equal to x	Prob(X ≤ 3)	0.8125
less than x	Prob(X < 3)	0.5
greater than or equal to x	Prob(X ≥ 3)	0.5
greater than x	Prob(X > 3)	0.1875

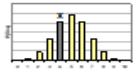
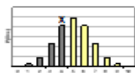
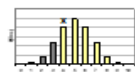
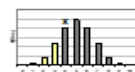
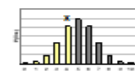
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There is a 31.26% probability that you land exactly 3 heads in 5 coin tosses?

Binomial Example

If I roll a dice 10 times what is the chance of getting exactly 3 of the same number ?

Binomial Distribution Probability Calculator			
Input the following information:			
Number of Trials	n		10
Event Probability	p		0.17
Number of Events in n Trials	x		3

Select the Probability of Interest from the following probabilities:		
equal to x		Prob(X = 3) 0.159983302
less than or equal to x		Prob(X ≤ 3) 0.925852753
less than x		Prob(X < 3) 0.765869451
greater than or equal to x		Prob(X ≥ 3) 0.234130549
greater than x		Prob(X > 3) 0.074147247

There is 16% chance of getting exactly same number 3 times out of 10 throws ?

Binomial Example

If I roll a dice 10 times what is the Probability of getting no more that 3 throws that land on the number 2 ?

 Binomial Distribution Probability Calculator		
Input the following information:		
Number of Trials	n	10
Event Probability	p	0.17
Number of Events in n Trials	x	3

Areas:

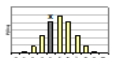
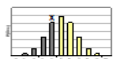
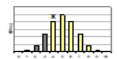
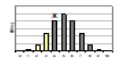
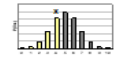
equal to x

less than or equal to x

less than x

greater than or equal to x

greater than x

Select the Probability of Interest from the following probabilities:		
	Prob(X = 3)	0.159983302
	Prob(X ≤ 3)	0.925852753
	Prob(X < 3)	0.765869451
	Prob(X ≥ 3)	0.234130549
	Prob(X > 3)	0.074147247

92.6% Probability of getting no more that 3 throws that land on the number 2?

Poisson Distribution

- The Poisson distribution measures the likelihood of a number of events occurring within a given time interval or within a given Surface Area or Volume.
- Events per hour, Errors per m², Defects per minute, Accidents per year

Basic assumptions:

- Discrete distribution (events are classified as success or failure)
- Length of the observation period (or area) is fixed in advance
- Occurrences are independent
- Events occur at a known average rate () λ

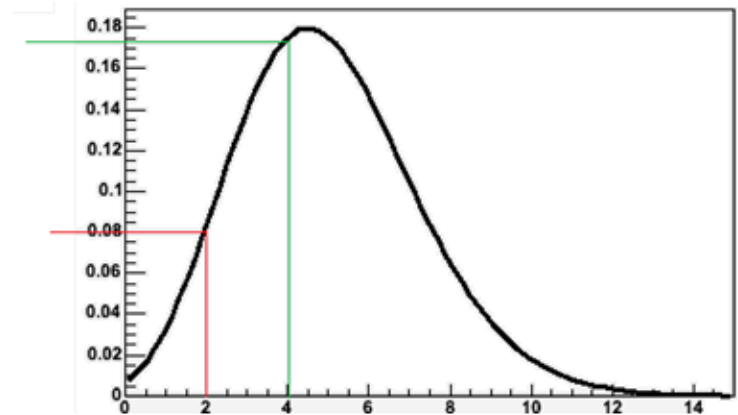
Examples of use:

- How many breakdowns occur within 3 months of a batch shipment
- What is the probability of 3 patients arriving in a waiting room between 12:00 & 13:00?
- What is the probability of having more than 10 defects over 100 m² of carpet ?

What Does Poisson Tell Us

Probability distribution function (PDF)

- The chance of **exactly** four success events occurring in a time interval is 17.5%
- The chance of **exactly** two success events occurring in a time interval is 8%

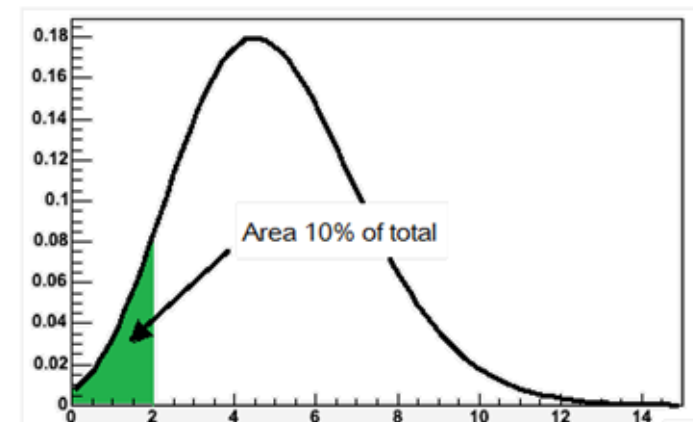


Cumulative distribution function (CDF)

- The chance of up to **(and including)** two success events or less in a time interval is about 10%

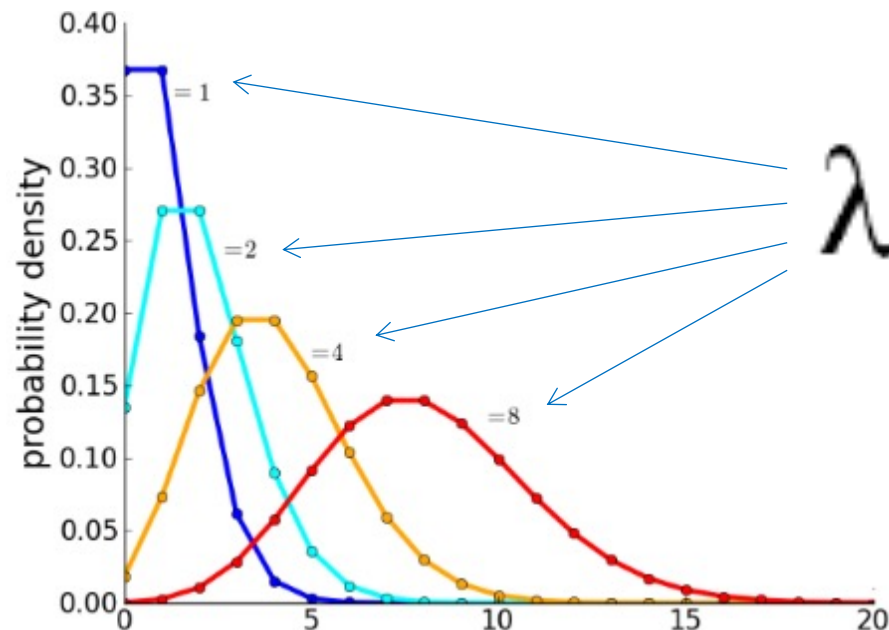
Inverse cumulative distribution function (ICDF)

- The largest value that will cover about 10% of my smallest success event combinations is two (or 80% of the time we will get two or more success events in the given time period)

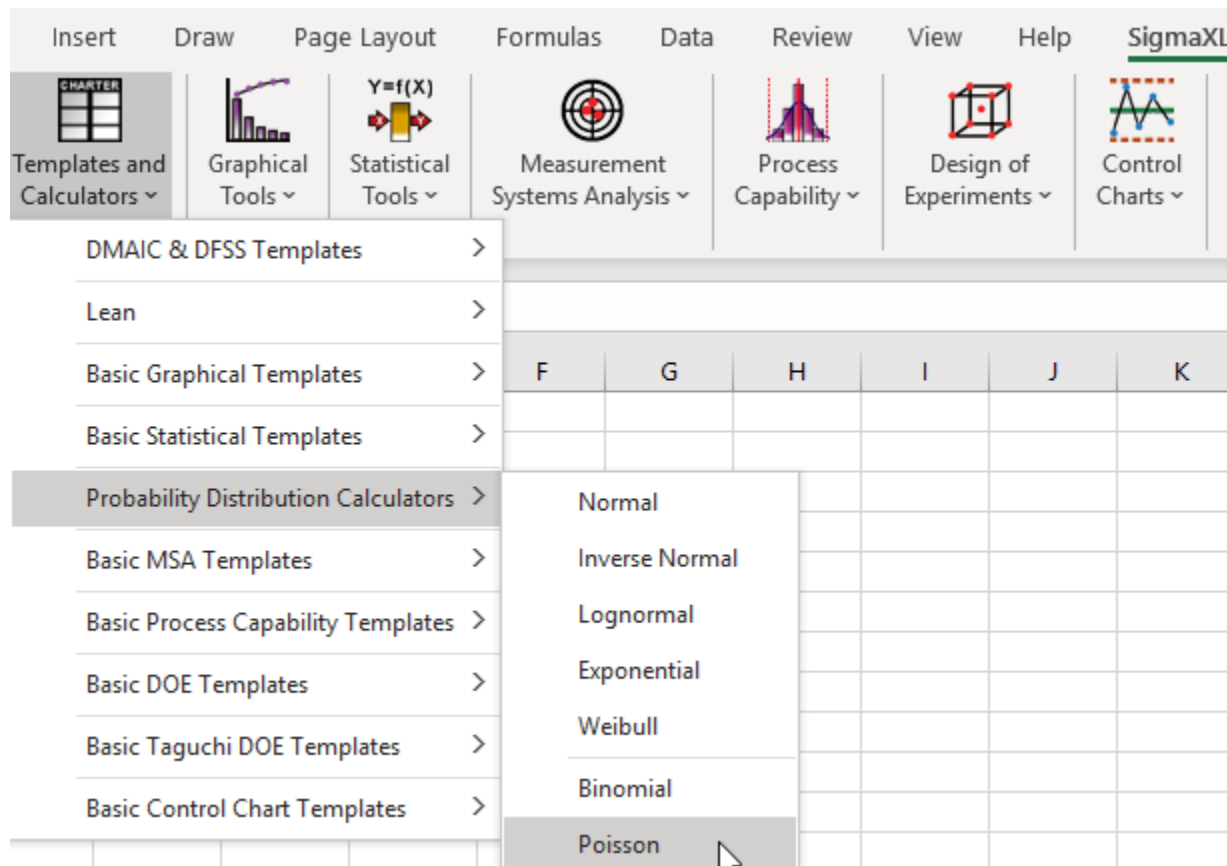


Poisson and Lambda

The key parameter for Poisson is the mean average number of events in the given interval. This parameter is called '**Lambda**' λ



Poisson Calculations



Poisson Calculations



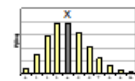
Poisson Distribution Probability Calculator

Input the following information:

Mean	λ	2
Number of Occurrences	x	1.5

Areas:

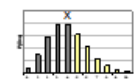
equal to x



Prob(X = 1.5)

0.270670566

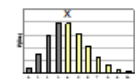
less than or equal to x



Prob(X ≤ 1.5)

0.40600585

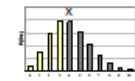
less than x



Prob(X < 1.5)

0.135335283

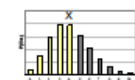
greater than or equal to x



Prob(X ≥ 1.5)

0.864664717

greater than x



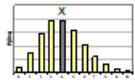
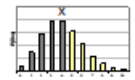
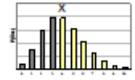
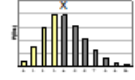
Prob(X > 1.5)

0.59399415

Binomial Example

The average number of customer complaints per day is 4. What is the probability of there being 5 complaints in 1 day ?

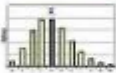
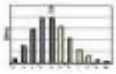
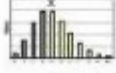
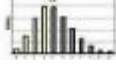
Sigma XL Poisson Distribution Probability Calculator		
Input the following information:		
Mean λ		4
Number of Occurrences x		5

Select the Probability of Interest from the following probabilities:		
Areas:		
equal to x	Prob(X = 5)	0.156293452
		
less than or equal to x	Prob(X ≤ 5)	0.785130387
		
less than x	Prob(X < 5)	0.628836935
		
greater than or equal to x	Prob(X ≥ 5)	0.371163065

There is 15.6% probability of there being 5 complaints in 1 day ?

Binomial Example

Patients arrive at a hospital 24 hour emergency room on average a a rate of 6 per hour ? What is the probability that in any 24 hour period you would have more than 160 arrivals ?

Sigma XL Poisson Distribution Probability Calculator		
Input the following information:		
Mean λ		144
Number of Occurrences x		160
Select the Probability of Interest from the following probabilities:		
Areas:		
equal to x		Prob(X = 160) 0.013370175
less than or equal to x		Prob(X ≤ 160) 0.91364806
less than x		Prob(X < 160) 0.900277884
greater than or equal to x		Prob(X ≥ 160) 0.099722116

There is 1 % probability of there being more than 160 arrivals in 1 day ?

Binomial Distribution

1. The probability of an electric component being defective is 0.00075 (0.075 %).
The components are supplied in boxes of 120 units.

What is the probability of getting exactly 1 defective in a box ?

What is the probability of getting at least 2 defective components in a box ?

2. Ten percent of computer parts produced by a certain supplier are defective.

What is the probability that a sample of 10 parts has no defective parts?

Binomial Distribution

3. Historically there is a 70 percent chance of a student passing a particular exam.

- a. What is the probability of 100% of students passing in a class of 10 ?
 - b. What is the probability of exactly 7 passing in a class of 10 ?
 - c. What is the probability of at least 7 passing in a class of 10 ?
-

4. The probability of a construction company winning a government work contract is 30% (the company won 6 contracts out of 20 it applied for over the past 2 years).

Find the probability of the company winning all 3 contracts out of the next 3 that it applies for. ?

Poisson Distribution

1. The number of typing mistakes made by a writer has a Poisson distribution. The mistakes are made independently at an average rate of 1.65 per page. Find the probability that a three-page letter contains no mistakes.

2. If the light bulbs in a house fail according to a Poisson law, and over the last 15 weeks there have been 5 failures, find the probability that there will either zero or one failure next week.

3. A fast food restaurant can expect two customers every 3 minutes, on average. What is the probability that four or fewer customers will enter the restaurant in a 9-minute period?

Poisson Distribution

4. A bank is interested in studying the number of people who use the Cash-machine located outside its office late at night. On average, 1.6 customers walk up to the Cash-machine during any 10-minute interval between 9pm and midnight.

a) What is lambda λ for this problem over a 10-minute period?

b) What is the probability of exactly 3 customers using the Cash-machine during any 10-minute interval?

5. The average number of homes sold by the London Estates Ltd company is 2 homes per week. What is the probability that more than 2 homes will be sold next week?

Measurement System Analysis

For Attribute Data

Attribute Measurement Error Analysis

Sometimes it is not convenient to measure the characteristics of interest numerically. For example how do you measure “must be comfortable to wear”?

In situations such as this, testing is opinion based and is subjective.

SigmaXL and some other stats packages offer tests for measurement systems based on subjective opinions

Is this Pizza really tasty ?



Attribute Measurement Error Analysis

- Attribute data is classifications rather than number measures
 - “Binary” data type is an example of Attribute data
- Unlike a numeric testing, an attribute test does not give information on the quality of a part but only indicates whether the part meets the stated criteria or not.
 - This decision can be based on a set of specified standards

Two attribute tests:

- Attribute Gauge R&R test
 - Attribute gage studies assess the amount of bias and repeatability of a gage when the response is a binary attribute variable.
- Attribute Agreement Analysis
 - (to assess the consistency and correctness of subjective ratings):
 - Within appraisers, when each appraiser rates each item more than once
 - Between appraisers
 - Against the standard, when you have a known rating for each part

Walkthrough of Attribute Agreement Analysis

A Food manufacturer decides to evaluate if a product is correctly wrapped on a 1-5 scale. Four members of the team volunteer to perform appraisals for the projects

Each makes two evaluations on each one of 30 sample sweets

A SigmaXL Gauge R&R worksheet is created to hold the data

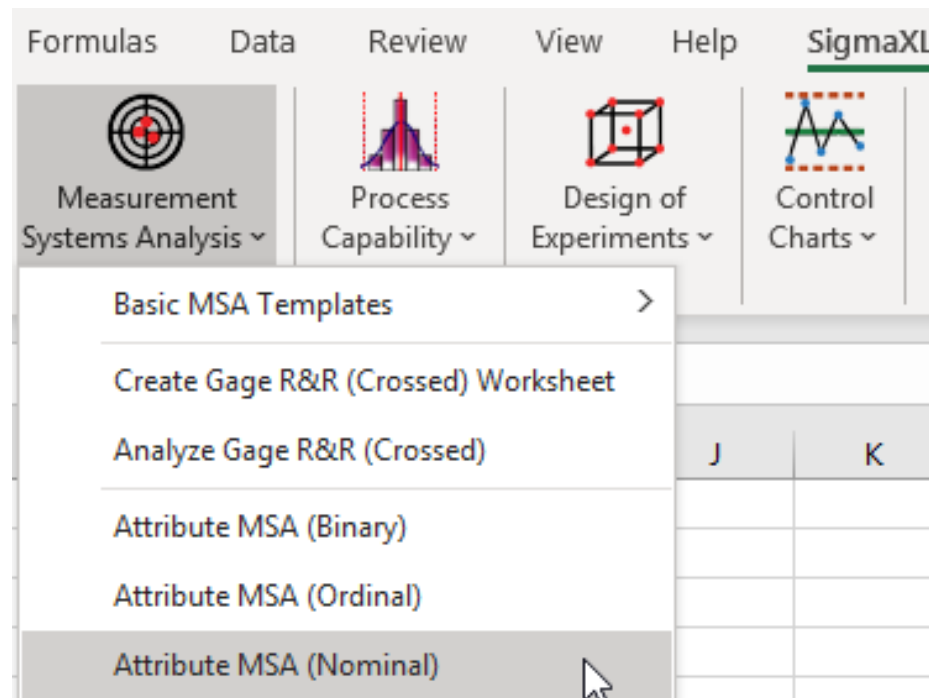
Because the correct value of each sample is known (this has previously been established) this data is entered into it's own column

A	B	C	D	E	F	G	H	I
Appraiser	Response	Sample	Trial	Standard				
Tom	5	1	1	5				
Tom	5	1	2	5				
Tom	3	2	1	3				
Tom	3	2	2	3				
Tom	1	3	1	1				
Tom	1	3	2	1				
Tom	2	4	1	2				
Tom	2	4	2	2				
Tom	1	5	1	1				

This worksheet is available in your SigmaXL Data folder – go to the MSA tab

Locating Attribute Agreement Analysis

SigmaXL → Measurement Systems Analysis → Attribute MSA (Nominal)



Configuring the Test

The appropriate columns are identified as is the column containing the known value (standard)

Attribute MSA (Nominal) ✕

Trial	<u>P</u> art/Sample >>	Sample	OK >> Cancel Help
	<u>A</u> ppraiser >>	Appraiser	
	<u>A</u> ssessed Result >>	Response	
	<u>T</u> rue Standard (Optional) >>	Standard	
	<< Remove		

☐ Report Information (Optional)	Confidence Level 95.0
Product/Unit Name: Performed By: Date: Notes:	Percent Confidence Interval Type: ☐ <u>W</u> ilson Score ☒ <u>E</u> xact

Text Output

- The results tab gives a detailed textual analysis of the investigation there are a number of categories:
 - Within appraisers
 - Each Appraiser vs Standard
 - Between Appraisers
 - All Appraisers vs Standard

Attribute Agreement Report:

Within Appraiser Agreement	# Inspected	# Matched	Percent	95.0% LC (Exact)	95.0% UC (Exact)	Fleiss' Kappa Overall Response
Lizzy	50	50	100.00	94.18	100.00	1.0000
Matt	50	43	86.00	73.26	94.18	0.8235
Phoebe	50	48	96.00	86.29	99.51	0.9496
Tom	50	45	90.00	78.19	96.67	0.8747

Each Appraiser vs. Standard Agreement	# Inspected	# Matched	Percent	95.0% LC (Exact)	95.0% UC (Exact)	Fleiss' Kappa Overall Response
Lizzy	50	47	94.00	83.45	98.75	0.9248
Matt	50	41	82.00	68.56	91.42	0.8616
Phoebe	50	46	92.00	80.77	97.78	0.9248
Tom	50	45	90.00	78.19	96.67	0.9373

Tests Used by SigmaXL

- So long as each appraiser assesses each unit more than once, Minitab will run several tests to investigate the consistency of each appraiser – between their own scores for each piece or unit, and between the appraisers as a group for each piece or unit
- The 2 most well known tests are:
 - **Fleiss' Kappa statistics**
 - **Kendall's Coefficient of Concordance**

Details on the next two slides ...

What Each Test Does

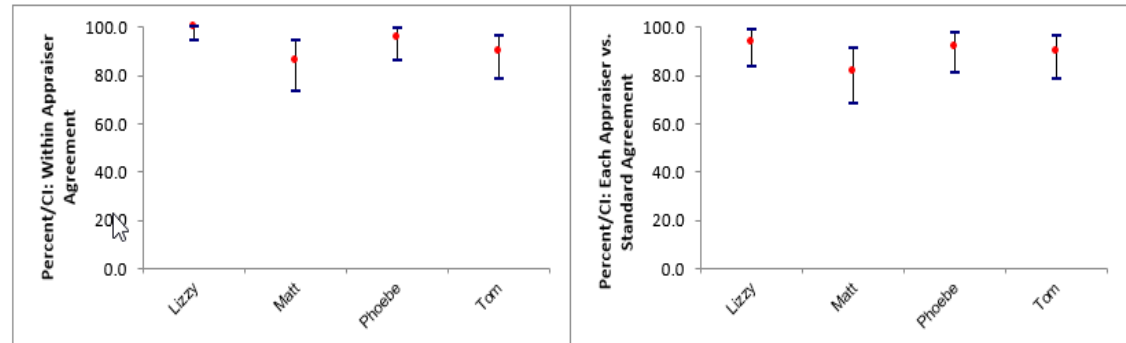
Test	What it does ...
Fleiss' Kappa Statistics	Measures agreement between appraisers Group consistency Tests whether the appraisers agree with each other when making an assessment: Ho: Any agreement between appraisers is due to random chance Ha: Any agreement between appraisers is NOT likely to be due to random chance
Kendall's Correlation Coefficient	Measures individual consistency Tests whether individual appraisers constantly assess each item the same: Ho: There is no association among multiple ratings made by an appraiser Ha: The ratings are associated with one another

“Indicators” Used by the Tests

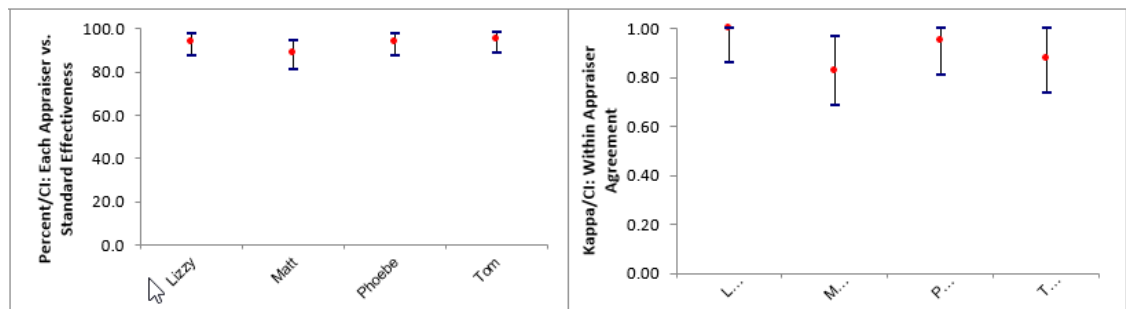
Test	What it does ...
Fleiss' Kappa value	If kappa = 1, then there is perfect agreement. If kappa = 0, the agreement is the same as would be expected by chance. Depending on the application, kappa less than 0.7 indicates that the measurement system needs improvement. kappa values greater than 0.9 are considered excellent. Look for a kappa value > 0.7. to check if the appraisers' rating consistency is within acceptable limits
Kendall's coefficient of concordance	Kendall's coefficient of concordance can range from 0 to 1. The higher the value of Kendall's, the less variance there is in an appraiser rankings (we are not saying they are correct only consistent)

Attribute Agreement - Graphical

- If you have multiple trials, then SigmaXL plots the percentage of assessment agreement within appraiser



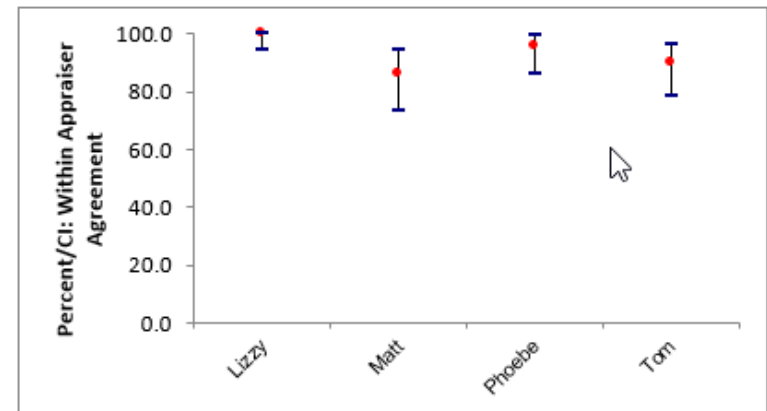
- If you have a known standard for each part, then the percent matched between appraiser and standard is plotted



See next slide for a description of this ...

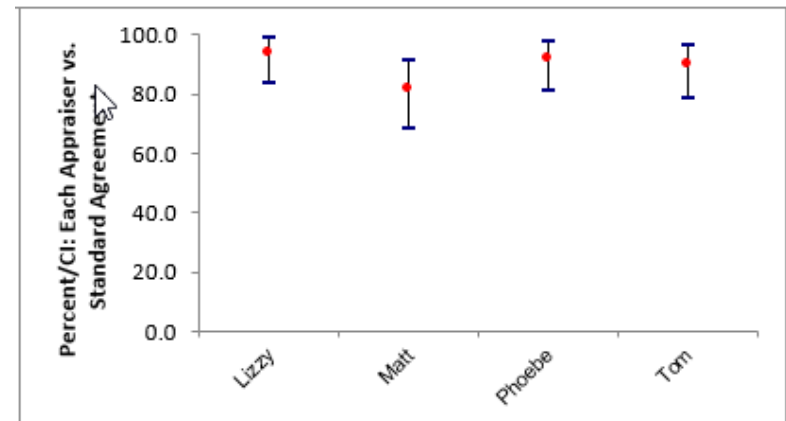
Graphs – Within Appraiser

- The Within Appraiser graphs show you the consistency of each operator's answers
- For each appraiser, the graph consists of the following:
 - Red circle, which provides the actual percent matched.
 - Blue line, which provides a 95.0% confidence interval for percent matched.
 - Blue lines, which provide the lower and upper limits for the 95.0% confidence interval, respectively.



Graphs – Appraiser vs Standard

- The Appraiser versus Standard graph shows the correctness of each operator's answers
- For each appraiser, the graph consists of the following:
 - Red circle, which provides the actual percent matched
 - Blue line, which provides a 95.0% confidence interval for percent matched
 - Blue lines, which provide the lower and upper limits for the 95.0% confidence interval, respectively



What Conclusions

Can we draw from this Attribute Measurement Error Analysis?

- The appraisers seem to be consistent in their measurements – both between the measurements they make and between themselves as a group
- They also seem to rate each piece correctly (when compared to an established standard)
- This accuracy is unlikely to have happened by chance
- So in conclusion it appears that the appraisers are doing a good job, they are accurate and precise in their ratings. There is variation between appraisers but not enough to cause concern.

Example of Attribute Agreement Analysis

Fabric appraisers at a textile printing company rate the print quality of cotton fabric on a 1 to 5 point scale. The Black Belt wants to assess the consistency and correctness of the appraisers' ratings. The Black Belt asks four appraisers to rate print quality on 50 samples of fabric twice, in random order.

Quality Scale 1  5



SigmaXL for Attribute Agreement Analysis

Open the sample data, TextilePrintQuality.xlsx.

1. Choose **Measurement Systems Analysis > Attribute MSA Nominal**
2. In **Part/Sample**, enter *Sample*.
3. In **Appraiser**, enter *Appraiser*.
4. In **True Standard**, enter *Standard*.
5. Click **OK**.

Answers

Binomial

1. a) 0.0823 or 8.23 % b) $1 - 0.9962 = 0.0038$ (0.38 %)
2. 0.349 (34,9 %)
3. a) 0.0282 2.28% b) 0.2668 26.7% c) 0.65 (65%)
4. 0.027 (2.7 %)

Its is VERY Unlikely it would win 3 out of 3 based on past performance !

Poisson

1. $\lambda = 1.65$ per page
 $\lambda = 4.95$ per three pages
Answer = 0.0071
2. $\lambda = 0.333$ Cumulative to 1 Answer = 0.9554
3. $\lambda = 2$ per 3 mins therefore $\lambda = 6$ per 9 minutes
Cumulative Prob up to 4 people with $\lambda = 6$ Answer = 0.285 or 28.5 %
4. a) $\lambda = 1.6$ b) Probability of 0.137828 or approx. 13.8%
5. $\lambda = 2$ Find Cumulative prob first then $1 - 0.676676 = 0.3233$ (32.33 %)

Good Luck With Your Black Belt Exam!

